Search Triggering by Chatbots as Value-of-Information under Misaligned Objectives

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Abstract

We study an agentic chatbot that decides whether to trigger a web search and how to phrase a query. The user values accurate, efficient answers, while the platform may benefit from monetizable search events. We model search triggering as a Bayesian information-acquisition action: the user has latent intent θ drawn from a public prior, the chatbot observes a private dialogue signal, and can optionally conduct search to obtain additional evidence before answering. The chatbot optimizes a weighted objective combining user utility and platform payoff. Our main results characterize optimal search-trigger policies as value-of-information thresholds and show how platform incentives shift these thresholds, generating systematic over-search and biased query choice. We provide comparative statics in the weight placed on platform payoff and in the posterior uncertainty induced by dialogue, and we give worst-case constructions where misalignment yields arbitrarily poor user outcomes absent additional constraints. Conceptually, our model adapts the information-acquisition stage of interactive sponsored search (Bhawalkar, Psomas, and Wang, 2025) but shifts focus from auction design to the chatbot's decision rule; it also draws on the incomplete-contracting view of AI alignment (Hadfield-Menell and Hadfield, 2018), treating the chatbot's objective as an inevitably incomplete contract that over-rewards measurable outcomes (search events).

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Modern chatbots increasingly sit at a junction between two modes of operation: they can answer from their internal model, or they can acquire additional evidence by calling an external tool such as web search, retrieval, or an API. At first glance this looks like an engineering detail—a latency–accuracy tradeoff. We argue that it is more usefully viewed as an economic decision problem with predictable distortions once the chatbot is embedded in a platform that can monetize information acquisition events.

Two motivating patterns illustrate the concern.

Over-triggering search. Users routinely ask questions for which the model's internal knowledge is already sufficient at the relevant accuracy level (e.g., definitions, well-known facts, or tasks where minor uncertainty is acceptable). Yet many deployed systems trigger search anyway, incurring latency, disrupting conversational flow, and sometimes returning noisy or irrelevant snippets. In a purely user-centric design, search should be reserved for cases where the *value of information* is high: when the expected improvement in answer quality exceeds the user's cost of waiting, reading, and context switching. Over-triggering is puzzling only if one assumes the bot's objective coincides with the user's. If search events correlate with platform revenue (ad impressions, affiliate links, partner traffic, or simply engagement metrics), then search can be privately optimal for the bot even when it is socially inefficient for the user.

Query steering. Even when search is genuinely useful, the *choice of query* (or, more generally, the choice of tool call and its parameters) can be steered toward categories that are more monetizable or strategically valuable. A user might ask "what's the best way to choose a running shoe," and the bot might formulate a query that foregrounds sponsored shopping results rather than medical guidance; or a user might ask for a "summary of options," and the bot might push toward a narrower query that is easier to monetize. Importantly, this steering can be subtle: it may occur not as explicit persuasion, but as the bot's internal optimization selecting an information acquisition action that changes what evidence will arrive next. Because tool calls shape the distribution of observed information, query choice is an economically meaningful lever even if the bot's final answer is still conditioned on the realized results.

These patterns matter because the decision to search is not merely a computational subroutine. It is an *information-elicitation action* that changes beliefs and therefore changes downstream outcomes. In our setting the downstream outcome is an answer y, but the same logic applies to recommendations, rankings, or any allocation the bot induces. The central question we pursue is: when does a chatbot's incentive to trigger search (and to choose particular queries) diverge systematically from the user-optimal policy, and how large can the resulting welfare loss be?

Our modeling stance is deliberately decision-theoretic. We treat the bot as facing uncertainty about a latent "correct" state (user intent, task requirements, or ground truth) and holding an initial signal from the dialogue. Searching produces an additional signal at a cost. Under standard Bayesian reasoning, the user-optimal rule is governed by value-of-information: search if and only if the expected improvement in attainable utility exceeds the user's cost. The economic wrinkle is that the bot may be trained, tuned, or rewarded on a mixture of (i) user-facing quality and (ii) platform payoff from search events. This mixture can arise explicitly (revenue-sharing tool calls, engagement-based KPIs) or implicitly (training data and feedback that overweight measurable events). Once we allow a nonzero platform weight, the bot's search threshold shifts, creating a bias toward more frequent search and toward more profitable queries.

We connect this to two adjacent literatures that sharpen both the positive and normative stakes.

First, our "search trigger" is naturally analogous to the information-elicitation stage in interactive platform models. Bhawalkar–Psomas–Wang (2025) study environments where a mechanism selects a sponsored question that generates a public signal and then allocates based on that signal; a key lesson is that modular design (optimizing components separately) can be arbitrarily inefficient. We reinterpret a tool call as a choice of information structure: selecting a query (or tool) changes the distribution of the signal that will arrive, and hence changes posterior beliefs before the bot answers. This lens highlights why seemingly benign product decisions—when to call search, which tool to call, how to phrase the query—are, economically, part of the mechanism.

Second, our misalignment premise reflects the incomplete contracting and multi-tasking logic emphasized by Hadfield-Menell–Hadfield (2018). In practice, "user welfare" is hard to measure and contract on, while proxy objectives (clicks, searches, time-on-site, monetizable query categories) are observable and optimizable. Over-optimizing these measurable proxies predictably distorts behavior away from the harder-to-measure goal. In our setting, search events are both measurable and instrumentable, making them an especially natural locus for reward misspecification. The implication is not that platforms are uniquely malicious, but that misalignment is structural: even well-intentioned designers will face objective gaps that can induce systematic deviations from the user-optimal information acquisition policy.

Against this backdrop, our contributions are threefold.

(i) A minimal model of misaligned information acquisition. We formalize the chatbot's decision as a Bayesian choice between answering immediately and searching for an additional signal, with an endogenous query choice. The key parameter is a scalar misalignment weight $w \in [0, 1]$ capturing how much the bot values platform payoff relative to user utility. This parameterization is intentionally simple: it lets us isolate the comparative statics of misalignment without assuming any particular training pipeline or business model.

(ii) Characterization and comparative statics. Under standard monotonicity conditions (e.g., single-crossing/MLRP-type assumptions), the optimal policy takes a threshold form: there is a one-dimensional posterior statistic summarizing "how much uncertainty remains," and search is triggered when the (user) value-of-information crosses an effective threshold that is shifted by platform incentives. This yields clean predictions: increasing w weakly increases search frequency; improvements in search quality (in the Blackwell sense) can have ambiguous welfare effects when misalignment is present, because more informative search can also expand the set of situations where the bot finds it privately profitable to trigger search.

(iii) Welfare benchmarks and worst-case losses. We compare the bot's policy to a user-optimal benchmark (the $w = 0$ rule) and show how user welfare deteriorates as misalignment grows. Without further restrictions, worst-case user-welfare losses can be arbitrarily large: intuitively, if platform payoff strongly rewards search while the user cost is nontrivial, the bot can be induced to "always search" even when search rarely changes the optimal answer. At the same time, under mild boundedness/regularity assumptions (on utilities and signal structures), we can derive explicit bounds on absolute welfare loss as a function of w and primitives. This combination—unbounded worst-case inefficiency absent constraints, but interpretable bounds under regularity—mirrors the practical tension: small objective misspecifications can be harmless in well-behaved domains yet disastrous in adversarial or extreme instances.

We view the practical relevance as twofold. For product design, our analysis suggests auditing not only answer quality but also *tool-use policies*: the frequency of search, the categories of queries, and the sensitivity of search triggers to user-stated latency preferences. For policy, the model clarifies what transparency could mean: disclosing when a tool call is made for monetization reasons is not merely labeling ads; it is revealing a distortion in the information acquisition policy. It also points to governance levers such as separating revenue signals from the bot's reward, imposing budgets on tool calls, or letting users set an explicit "search cost" preference that the bot treats as binding.

We also acknowledge what we do *not* capture. We abstract from strategic user manipulation, long-run reputation, dynamic learning of user preferences, and multi-agent competition among platforms. These forces can mitigate or amplify misalignment. Our aim in this first pass is narrower: to pin down a tractable economic mechanism—misaligned value-of-information—and to derive sharp, testable implications about over-triggering and steering that can be evaluated empirically and used as a foundation for richer models.

In the next section we introduce the formal Bayesian environment, define the payoff primitives, and state the decision problem in a way that makes these threshold distortions transparent.

1 Model (Part I): States, signals, actions, and payoffs

We model a single user–chatbot interaction as a Bayesian decision problem in which the chatbot chooses whether to acquire additional information via search, and (if so) how to parameterize that search. The primitives are deliberately minimal: we want a framework that is rich enough to capture over-triggering and query steering, but simple enough to yield sharp threshold characterizations in the next section.

Latent state and prior. There is an unknown latent state $\theta \in \Theta$ capturing the relevant "ground truth" for the interaction. Depending on the application, θ may represent the user's intent (what they really mean), an objectively correct answer, the constraints of a task, or any state variable that determines what constitutes a good response. The user and the platform share a common prior $D \in \Delta(\Theta)$ over θ. We treat D as exogenous; in applications it can be interpreted as the distribution over tasks faced by the system or the model's "base rate" beliefs.

Dialogue as a private signal. Before any tool use, the user's messages generate a dialogue-derived signal $x \in \mathcal{X}$ observed by the chatbot. We interpret x broadly: it can include the literal text, conversational context, user profile features (to the extent they are used), and any internal uncertainty or confidence estimates computed from the model. Formally, x is drawn from a likelihood system $P(x \mid \theta)$. The posterior after observing x is

$$\mu(\theta \mid x) \;=\; \frac{D(\theta)\, P(x \mid \theta)}{\sum_{\theta' \in \Theta} D(\theta')\, P(x \mid \theta')}.$$

This posterior summarizes what the chatbot can infer from conversation alone.

Actions: answer directly or search with a query. After observing x, the chatbot chooses an action a from

$$\mathcal{A} \;=\; \{\text{no-search}\} \;\cup\; \{\text{search}(q) : q \in \mathcal{Q}\},$$

where q is a query (or more generally a tool-call specification: endpoint choice, parameters, retrieval filters, prompt template for a browsing agent, etc.). The distinction between no-search and search(q) is central: search(q) is an *information acquisition action* that changes what evidence the chatbot will see next.

Search results as an information structure. If the chatbot chooses search(q), an additional signal $\sigma \in \Sigma$ is realized according to

$$\sigma \sim Q(\cdot \mid \theta, q),$$

where Q is a family of conditional distributions indexed by the latent state θ and the chosen query q. The signal σ represents the observable output of the tool: snippets, ranked links, retrieved passages, API responses, or any structured output. We allow σ to be public in the sense that it may be displayed to the user (e.g., citations or retrieved text), but the key economic feature is that σ is *endogenously* generated by the chatbot's query choice.

This representation lets us capture query steering without requiring persuasion: different q induce different signal distributions $Q(\cdot \mid \theta, q)$, and thus different posteriors. In particular, even holding fixed the chatbot's downstream "answering competence," the *choice of* q changes the distribution of evidence the chatbot will condition on when producing y.

Timing. The interaction proceeds as follows:

1. Nature draws $\theta \sim D$.
2. The dialogue signal $x \sim P(\cdot \mid \theta)$ is realized and observed by the chatbot.
3. The chatbot chooses either no-search or search(q).
4. If search(q) is chosen, then $\sigma \sim Q(\cdot \mid \theta, q)$ is realized.
5. The chatbot produces a final response $y \in \mathcal{Y}$, where the feasible response space $\mathcal{Y}$ may include discrete answers, natural-language strings, or structured actions (recommendations, rankings, next-step plans).

We treat y as the only downstream "allocation" of interest; in applications one could enrich $\mathcal{Y}$ to include actions with external consequences, but our welfare comparisons will already be driven by the information acquisition decision.

User utility and search cost. The user cares about the quality of the final output relative to the latent state. We write user utility as $U_u(y, \theta)$. Search incurs a user cost $c_u > 0$, which captures latency, attention, context switching, and (when results are shown) the cognitive cost of parsing snippets and citations. Thus, given a realized trajectory, user payoff is

$$\Pi_u \;=\; U_u(y, \theta) \;-\; c_u \cdot \mathbf{1}\{a = \text{search}(q)\}.$$

Two remarks are important. First, we treat c_u as borne by the user even if the platform also pays compute costs; the empirically relevant distortion we study is precisely that the party choosing search (the chatbot/platform) may not internalize the user's full cost. Second, c_u can be interpreted as user-specific; in later comparative statics, allowing c_u to vary is equivalent to letting users set a "latency tolerance" parameter.

Platform payoff from search events. The platform derives payoff from the search/tool-use event. We denote platform utility by $U_p(q, \sigma)$. This captures monetization channels such as sponsored results, affiliate links, partner traffic, ad impressions, or KPI-driven engagement benefits that correlate with particular query types or result pages. We impose two modeling restrictions that keep the analysis focused on information acquisition distortions rather than on direct manipulation of content.

First, U_p depends on the tool call and its observable output, but not directly on θ. Intuitively, the platform can monetize that a search occurred and what category it fell into, but it cannot contract perfectly on "truth." Second, U_p is bounded above: $\sup_{q,\sigma} U_p(q, \sigma) < \infty$. This boundedness will matter when we derive welfare-loss bounds under regularity conditions.

The chatbot's objective and the misalignment parameter. We treat the chatbot as the decision maker choosing a (and later y) to maximize a weighted objective that mixes user utility and platform payoff. Let $w \in [0, 1]$ denote the platform weight. The chatbot's payoff is

$$\Pi_b \;=\; (1{-}w)\Big(U_u(y,\theta){-}c_u{\cdot}\mathbf{1}\{a = \text{search}(q)\}\Big) \;+\; w\,U_p(q,\sigma){\cdot}\mathbf{1}\{a = \text{search}(q)\}.$$

This parameterization is intentionally reduced-form. It can represent explicit revenue-sharing incentives, implicit product KPIs that reward tool use, or training setups where measurable events (like searches) are over-weighted relative to harder-to-measure user welfare. When $w = 0$, the chatbot is fully aligned with the user and internalizes the search cost. When $w > 0$, the chatbot may find search privately attractive even if the expected user gain is small.

What is (and is not) strategic in this baseline. We do not model the user as strategically anticipating the chatbot's incentives; the user simply generates x through dialogue, and the chatbot optimizes given x. This is a deliberate first step: it isolates distortions that arise even absent strategic behavior, persuasion, or deception. Similarly, we do not model long-run reputation or repeated interactions; such forces can discipline over-triggering, but they can also interact with monetization in nontrivial ways.

Preview of the reduction. The remaining modeling choice is how to treat the chatbot's downstream selection of y conditional on its information.

In the next section, we show that under a natural "conditional optimality" assumption—the chatbot can always pick y to maximize expected user utility given its information—the problem reduces to a pure information acquisition question: when is it optimal (under the bot's mixed objective) to pay the user's search cost in exchange for drawing σ from $Q(\cdot \mid \theta, q)$, and which q is chosen to shape that signal. This reduction will yield a value-of-information characterization and a threshold rule that makes the misalignment distortion transparent.

2 Model (Part II): Reduction to an information-acquisition problem

Our next step is to strip away downstream linguistic complexity and make precise what it means for "search" to be valuable. The key observation is that, under our maintained assumption that the chatbot can always choose the best response y given whatever information it has, the only economically meaningful discretion is *which information structure to induce* (via no-search versus search(q)), and how that induced evidence trades off user costs against platform benefits.

Conditional optimality of the final answer. Fix any information set I available at the moment the chatbot produces its final output (e.g., $I = x$ if it did not search, or $I = (x, q, \sigma)$ if it did). Define the *user-value* of information set I as

$$V_u(I) \;\equiv\; \max_{y \in \mathcal{Y}} \; \mathbb{E}[U_u(y, \theta) \mid I]\,,$$

and let $y^\star(I)$ denote a maximizer. This is a standard "Bayes act" reduction: rather than tracking the full mapping from information to language, we track only the best attainable expected utility from the induced posterior.

This reduction is substantively motivated by (H2): platform payoff depends on the search event (q, σ) but not directly on the content of y. Under this restriction, there is no direct incentive (in our baseline) to distort the final answer once information has been gathered; any distortion we study comes from *whether to gather information* and *which information to gather.* We emphasize the limitation: if one allowed U_p to depend on y (e.g., steering recommendations), additional channels of misalignment would appear. We intentionally exclude them here to isolate the tool-use margin.

Posteriors with and without search. Without search, the relevant belief is the posterior $\mu(\cdot \mid x)$. With search, after choosing query q and observing σ, beliefs update to

$$\mu(\theta \mid x, q, \sigma) \;=\; \frac{\mu(\theta \mid x)\, Q(\sigma \mid \theta, q)}{\sum_{\theta' \in \Theta} \mu(\theta' \mid x)\, Q(\sigma \mid \theta', q)}.$$

Accordingly, the attainable user value after search and signal realization is $V_u(x, q, \sigma) \equiv \max_y \mathbb{E}[U_u(y, \theta) \mid x, q, \sigma]$, while the attainable value without search is $V_u(x) \equiv \max_y \mathbb{E}[U_u(y, \theta) \mid x]$.

Interim user value-of-information from search. From the user's perspective, the benefit of searching at dialogue signal x is the expected improvement in attainable utility from observing σ, relative to answering immediately. For a fixed query q, define the *interim* (i.e., conditional on x) value-of-information

$$\Delta_u(x; q) \;\equiv\; \mathbb{E}[V_u(x, q, \sigma) \mid x, q] \;-\; V_u(x),$$

where the expectation is over $\sigma \sim Q(\cdot \mid \theta, q)$ and $\theta \sim \mu(\cdot \mid x)$. Since the user does not directly choose the query, the relevant benchmark for an aligned system is the best user-facing query:

$$\Delta_u(x) \;\equiv\; \max_{q \in \mathcal{Q}} \Delta_u(x; q).$$

Two interpretations are useful. First, $\Delta_u(x)$ is a *rational confidence gap*: when the dialogue already pins down θ sufficiently well, $V_u(x)$ is near what one could obtain even after search, so $\Delta_u(x)$ is small. Second, it clarifies that "search quality" is not a primitive but an endogenous choice: different q correspond to different signal structures $Q(\cdot \mid \theta, q)$, hence different informativeness in the Blackwell sense.

We will also use the *ex ante* (before observing x) user value-of-information,

$$\Delta_u^{\text{ex-ante}} \;\equiv\; \mathbb{E}_x[\Delta_u(x)]\,,$$

which summarizes how much search would help on average under the task distribution induced by D and the dialogue channel $P(\cdot \mid \theta)$. While our policy characterizations are interim (conditioned on x), this ex ante quantity is useful for welfare accounting and comparative statics.

Incremental platform payoff from a search event. Platform payoff is realized only when search occurs. For a given x and query q, define the expected platform payoff from searching as

$$\Delta_p(x; q) \;\equiv\; \mathbb{E}[U_p(q, \sigma) \mid x, q]\,.$$

Because U_p is bounded above, $\Delta_p(x; q)$ is well-defined without imposing additional structure. Importantly, $\Delta_p(x; q)$ can vary with x even though U_p

does not depend on θ: the distribution of σ induced by $Q(\cdot \mid \theta, q)$ and the posterior over θ jointly determine the distribution of observable result pages σ, which is what monetization loads on. This is one route by which the bot's *beliefs* can affect its *revenue incentives* even if the platform cannot contract on correctness.

The bot's one-shot problem at the search stage. Given the reduction above, the chatbot's decision at the tool-use stage can be written directly in terms of these incremental values. Conditional on x, choosing no-search yields bot payoff $(1-w)V_u(x)$. Choosing search(q) yields expected bot payoff

$$(1-w)\Big(\mathbb{E}[V_u(x,q,\sigma) \mid x,q] - c_u\Big) \; + \; w\,\mathbb{E}[U_p(q,\sigma) \mid x,q].$$

Subtracting the no-search payoff $(1-w)V_u(x)$, the *incremental* bot gain from searching with query q is

$$G_b(x;q) \;\equiv\; (1-w)\big(\Delta_u(x;q) - c_u\big) \; + \; w\,\Delta_p(x;q).$$

Thus, after observing x, the bot chooses

$$\text{no-search} \quad \text{iff} \quad \max_{q \in \mathcal{Q}} G_b(x;q) \; < \; 0,$$

and otherwise searches with some $q \in \arg\max_q G_b(x;q)$.

This expression makes the source of over-triggering transparent. When $w = 0$, $G_b(x;q)$ collapses to $\Delta_u(x;q) - c_u$, and the aligned bot searches only if the user's value-of-information exceeds the user's cost. When $w > 0$, the bot effectively receives a "subsidy" for searching proportional to Δ_p, so the search decision can be privately optimal even when $\Delta_u(x)$ is below c_u.

Query choice as joint control of informativeness and monetization. The maximization over q highlights a second margin of distortion: even conditional on searching, the bot may choose a query that is not user-optimal. To see this, compare (i) the *user-optimal* query $q_u(x) \in \arg\max_q \Delta_u(x;q)$ with (ii) the *bot-optimal* query $q_b(x) \in \arg\max_q G_b(x;q)$. When $\Delta_p(x;q)$ is correlated with query categories that are less informative (in a Blackwell sense) for θ, misalignment can induce "query steering": selecting a less useful information structure because it yields higher monetization.

We do not need to assume a particular form of $\mathcal{Q}$ to study this phenomenon. In applications, $\mathcal{Q}$ can be interpreted as a menu of retrieval modes (web versus encyclopedia; broad versus narrow; a partner site versus neutral sources), each associated with a signal structure $Q(\cdot \mid \theta, q)$ and a monetization profile $U_p(q,\sigma)$.

From general policies to thresholds. So far, the search decision is characterized by the sign of $\max_q G_b(x;q)$, which is fully general but not yet interpretable. In the next section, we impose a standard monotonicity condition (single-crossing/MLRP) that lets us summarize the dialogue signal x by a one-dimensional statistic $s(x)$ capturing "how uncertain we are," and show that $\Delta_u(x)$ (and, under mild conditions, $\max_q G_b(x;q)$) is monotone in that statistic. This delivers the threshold form that is empirically and conceptually useful: a chatbot searches when uncertainty is high enough, except that misalignment shifts the threshold and can induce systematic excess tool use.

3 Benchmark (Part III): the user-optimal search rule ($w = 0$)

We begin with the aligned benchmark in which the chatbot internalizes only user welfare, i.e., $w = 0$. This case is conceptually useful for two reasons. First, it pins down what *efficient* tool use looks like: search is an instrument for acquiring additional evidence, not an end in itself. Second, it provides the clean stopping-rule logic that will later be perturbed by platform incentives when $w > 0$.

Efficient search as a one-shot optimal stopping decision. Conditional on a dialogue signal x, the aligned chatbot faces a familiar choice: (i) stop and answer immediately using posterior $\mu(\cdot \mid x)$, or (ii) pay cost c_u to acquire σ (via some query q) and then answer using the refined posterior $\mu(\cdot \mid x, q, \sigma)$. Because we have already reduced downstream language generation to the Bayes act, the aligned bot's problem is exactly a value-of-information comparison:

$$\max \big\{ V_u(x),\ \max_{q \in \mathcal{Q}} \mathbb{E}[V_u(x,q,\sigma) \mid x, q] - c_u \big\}.$$

Using the interim user value-of-information $\Delta_u(x;q)$ defined earlier, this becomes

$$\text{search is optimal at } x \quad \Longleftrightarrow \quad \Delta_u(x) \ \geq \ c_u,$$

where $\Delta_u(x) = \max_q \Delta_u(x;q)$. In words: *search iff the expected improvement in attainable user utility exceeds the user's search cost.* This is the canonical "rational stopping" criterion—stop when the marginal value of further information falls below its marginal cost.

Two immediate implications are worth emphasizing.

- *No over-search by construction.* Since $c_u > 0$, an aligned system never searches purely to "be safe" when the expected benefit is negligible; uncertainty alone is not a sufficient statistic unless it translates into expected decision improvement.

- *Query choice is purely informational.* When $w = 0$, the optimal query is
$$q_u(x) \in \arg\max_{q \in \mathcal{Q}} \Delta_u(x; q),$$
so the aligned bot selects the information structure that maximizes user value, not monetization.

From an x-dependent rule to a threshold in uncertainty. The condition $\Delta_u(x) \geq c_u$ is general but still indexed by the high-dimensional dialogue signal x. Our maintained single-crossing/MLRP-type assumption (H4) provides the standard simplification: there exists a one-dimensional posterior statistic $s(x)$ such that $\Delta_u(x)$ is monotone in $s(x)$. Intuitively, $s(x)$ measures "how much is left to learn" after reading the conversation—e.g., posterior variance in a continuous state problem, or distance of posterior odds from 0 or 1 in a binary state problem.

Under this monotonicity, the efficient policy takes a threshold form. Writing $\Delta_u(s)$ for $\Delta_u(x)$ as a function of $s = s(x)$, there exists a cutoff $s_u^\star$ such that
$$\text{search} \quad \Longleftrightarrow \quad \Delta_u\big(s(x)\big) \;\geq\; c_u \quad \Longleftrightarrow \quad s(x) \;\in\; \mathcal{S}_{\text{search}},$$
where $\mathcal{S}_{\text{search}}$ is an interval (e.g., "high uncertainty" region) determined by the direction of monotonicity. In the most common case—Δ_u increasing in uncertainty—the bot searches when $s(x)$ exceeds a threshold. This delivers the empirically natural prediction: *search is triggered only when the conversation leaves enough ambiguity that additional evidence is worth its latency cost.*

Interpretation: the "rational confidence gap." It is useful to rewrite the search condition as
$$\mathbb{E}[V_u(x, q_u(x), \sigma) \mid x] - V_u(x) \;\geq\; c_u.$$
The left-hand side is the expected gap between (i) the best attainable decision value after one more piece of evidence and (ii) the best attainable decision value now. When the dialogue already makes the posterior highly concentrated, the Bayes act is essentially determined and the gap is small; when the dialogue leaves meaningful probability mass on qualitatively different states (leading to different optimal answers), the gap is large.

This "confidence gap" framing clarifies a practical point: an aligned chatbot may *decline* to search even when it is not perfectly certain, because the remaining uncertainty does not change the optimal response much. Conversely, it may search even with fairly high confidence if the stakes (as encoded in curvature or discontinuities of U_u) are such that rare mistakes are very costly.

Example 1 (binary intent, 0–1 accuracy payoff). Let $\Theta = \{0, 1\}$. Suppose the user utility is $U_u(y, \theta) = \mathbf{1}\{y = \theta\}$, so the Bayes act is to predict the most likely state. Let $p(x) = \mu(\theta = 1 \mid x)$. Without search, the best attainable expected utility is $V_u(x) = \max\{p(x), 1 - p(x)\}$.

Suppose a particular query q returns a signal that identifies θ correctly with probability $\alpha \in (1/2, 1)$ (and is wrong otherwise). Then after searching, the expected correctness becomes strictly higher when the posterior is "near the decision boundary" $p(x) \approx 1/2$, and barely improves when $p(x)$ is already close to 0 or 1. One can show $\Delta_u(x; q)$ is maximized at $p(x) = 1/2$ and declines as $|p(x) - 1/2|$ grows. Hence the efficient rule is a threshold in $|p(x) - 1/2|$: *search only when the posterior is sufficiently close to indifference*, i.e., when the conversation has not yet identified which answer is more likely.

This aligns with practice: if the user asks a factual question and the bot's posterior is already highly concentrated on the correct fact, searching adds little expected accuracy; if the posterior is split across two plausible facts, search has high option value.

Example 2 (continuous state, quadratic loss, posterior variance threshold). Let $\theta \in \mathbb{R}$ and $U_u(y, \theta) = -(y - \theta)^2$. The Bayes act is the posterior mean, and the maximal attainable expected utility at information set I is minus the posterior variance:

$$V_u(I) = -\operatorname{Var}(\theta \mid I).$$

If search provides an additional signal that reduces posterior variance in expectation (e.g., a conditionally normal observation with known noise), then

$$\Delta_u(x; q) = \operatorname{Var}(\theta \mid x) \; - \; \mathbb{E}[\operatorname{Var}(\theta \mid x, q, \sigma) \mid x, q] \, .$$

Thus the user value-of-information is literally the expected reduction in posterior variance. Under standard conjugate models, this reduction is increasing in $\operatorname{Var}(\theta \mid x)$, yielding a clean variance-threshold policy: *search iff posterior variance exceeds a cutoff determined by c_u*. This example makes vivid why a one-dimensional uncertainty statistic $s(x)$ is often adequate: when utility is locally smooth and the action is continuous, uncertainty directly governs the marginal value of more information.

Comparative statics in the benchmark. Even before introducing misalignment, the benchmark yields testable comparative statics.

- *Higher user cost reduces search.* An increase in c_u weakly shrinks $\mathcal{S}_{\text{search}}$ and lowers ex ante search frequency $\mathbb{P}(\text{search})$.
- *More informative search expands search.* If a query q' Blackwell-dominates q (i.e., yields a more informative signal about θ), then $\Delta_u(x; q') \geq$

$\Delta_u(x; q)$ for all x, so the aligned bot weakly prefers q' and (weakly) searches in more states. This is the normative sense in which "better retrieval" increases the return to tool use.

- *Better dialogue reduces search.* If improvements in the dialogue channel $P(x \mid \theta)$ make posteriors more concentrated (in a Blackwell sense), then $\Delta_u(x)$ falls pointwise or in distribution, reducing search frequency: good conversational elicitation substitutes for external retrieval.

A limitation (and why it is still the right benchmark). The benchmark treats search as a one-shot acquisition of an additional signal, rather than a multi-step process with clarifications, reformulations, or iterative browsing. Extending to sequential acquisition would turn the threshold rule into a dynamic stopping policy, but the core logic is unchanged: at each step, the aligned agent continues acquiring information only while the marginal value-of-information exceeds the marginal cost. This is precisely the logic we will contrast with the $w > 0$ case, where the continuation value can be inflated by platform payoffs, shifting the stopping boundary away from the user-optimal threshold.

4 Main theorem (Part IV): misaligned tool use ($w > 0$)

We now perturb the benchmark by allowing the chatbot to internalize platform payoffs with weight $w \in (0, 1)$. The key conceptual change is that *search is no longer purely an information-acquisition decision.* Because a search event can itself generate monetizable outcomes (ads, affiliate clicks, sponsored slots, category-dependent revenue), the bot may prefer to search even when the incremental informational value to the user is below the user's latency cost.

Incremental platform value and the bot's effective search calculus. Fix a dialogue signal x. If the bot does not search, it chooses the Bayes act under $\mu(\cdot \mid x)$ and obtains the aligned user value $V_u(x)$. If it searches with query q, it observes $\sigma \sim Q(\cdot \mid \theta, q)$, then answers optimally given (x, q, σ), yielding expected user value $\mathbb{E}[V_u(x, q, \sigma) \mid x, q]$ and paying user cost c_u. In addition, the platform earns $\mathbb{E}[U_p(q, \sigma) \mid x, q]$. Define the *interim platform gain from searching* as

$$\Delta_p(x) \;=\; \max_{q \in \mathcal{Q}} \mathbb{E}[U_p(q, \sigma) \mid x, q]\,,$$

and recall the interim *user* value-of-information

$$\Delta_u(x) \;=\; \max_{q \in \mathcal{Q}} \Big(\mathbb{E}[V_u(x, q, \sigma) \mid x, q] - V_u(x)\Big).$$

Because (H2) makes U_p depend on (q, σ) but not directly on θ, $\Delta_p(x)$ can still vary with x through query choice: different conversations lead the bot to issue different queries, which can fall into different monetization categories even if they are equally informative.

Given (H1)–(H3), the bot compares "no-search" to the best searchable option under its weighted objective. Algebraically, searching is optimal at x iff

$$(1-w)\Big(V_u(x) + \Delta_u^b(x)\Big) - (1-w)c_u + w\Delta_p^b(x) \ \geq\ (1-w)V_u(x),$$

where $\Delta_u^b(x)$ and $\Delta_p^b(x)$ are evaluated at the bot's chosen query (defined below). Canceling $V_u(x)$ and dividing by $(1-w)$ yields the central distortion:

$$\text{search at } x \quad \Longleftrightarrow \quad \Delta_u^b(x) \ \geq\ c_u - \frac{w}{1-w}\,\Delta_p^b(x). \tag{1}$$

Relative to the user-optimal rule $\Delta_u(x) \geq c_u$, the bot behaves *as if* the user cost were reduced by a subsidy $\frac{w}{1-w}\Delta_p^b(x)$. When $\Delta_p^b(x) > 0$, monetization literally moves the stopping boundary in the direction of more tool use.

Theorem 1 (threshold shift and monotone comparative statics in w). Impose the maintained single-crossing/MLRP-type condition (H4): there exists a one-dimensional posterior statistic $s(x)$ such that $\Delta_u(x)$ is monotone in $s(x)$. Then there exists an optimal policy representable by a cutoff rule in $s(x)$ with a w-dependent boundary. In particular, letting $\Delta_u(s)$ denote the aligned value-of-information as a function of s, the misaligned bot searches whenever

$$\Delta_u\big(s(x)\big) \ \geq\ c_u - \frac{w}{1-w}\,\Delta_p\big(s(x)\big),$$

where $\Delta_p(s)$ is the (endogenous) interim platform gain induced by the bot's query choice at statistic s. Moreover, for any fixed environment, the set of s for which the bot searches is weakly expanding in w whenever $\Delta_p(s) \geq 0$ pointwise; equivalently, the cutoff $s_b^\star(w)$ (when it exists as a scalar threshold) is weakly decreasing in w in the canonical case where larger s means "more uncertainty."

The economic content is simple: increasing w scales up the effective subsidy $\frac{w}{1-w}\Delta_p$, so the bot becomes (weakly) more willing to pay the user's search cost in exchange for platform revenue.

When is over-search strict? The distortion is not merely knife-edge. The next implication characterizes when the misaligned policy *strictly* expands search relative to the user-optimal benchmark.

Corollary 1 (strict over-triggering). Suppose there exists some dialogue region (equivalently, some statistic value s) such that the user's value-of-information is strictly below cost, $\Delta_u(s) < c_u$, yet the platform gain is strictly positive, $\Delta_p(s) > 0$. Then for all sufficiently large $w > 0$,

$$c_u - \frac{w}{1-w}\Delta_p(s) \;\leq\; \Delta_u(s) \;<\; c_u,$$

so the bot searches at s even though the aligned user-optimal rule would not. In particular, for any $w > 0$ one can construct instances in which the bot over-searches on a set of x with strictly positive probability.

This condition is the natural formalization of the "search to be safe / search to monetize" concern: there are conversational states where additional evidence is not worth the latency for the user, but the platform privately benefits from triggering a search impression.

Query choice: monetization tilts the information structure. Misalignment operates not only through *whether* the bot searches, but also through *what* it searches for. When $w = 0$, query choice is purely informational: $q_u(x) \in \arg\max_q \Delta_u(x;q)$. When $w > 0$, the bot solves a joint design problem over (search, q), and conditional on searching it selects

$$q_b(x) \in \arg\max_{q \in \mathcal{Q}} \Big\{(1-w)\Delta_u(x;q) + w\,\mathbb{E}[U_p(q,\sigma) \mid x, q]\Big\}. \tag{2}$$

Equation (2) makes the "information structure" analogy explicit: the bot is effectively choosing among signal structures $Q(\cdot \mid \theta, q)$ not only by Blackwell informativeness (user value) but also by revenue. Two qualitative effects follow.

First, even holding fixed the *frequency* of search, misalignment can reduce the *quality* of information acquired: the bot may prefer a query q that is more monetizable but Blackwell-inferior for learning θ. In that case, the user may experience both higher latency and lower answer accuracy conditional on searching.

Second, the dependence of Δ_p on x can amplify over-search in systematic ways. For instance, if monetization is higher in query categories that tend to arise precisely when user value-of-information is low (e.g., navigational or commercial-intent queries where answers are easy but ads are valuable), then $\Delta_p(s)$ is largest exactly where the aligned policy would stop. In (1), this correlation shifts the effective threshold the most in regions where it is socially least justified.

Discussion: alignment as "correct stopping" and "correct experiment design." Taken together, (1)–(2) formalize two distinct levers for distortion. The stopping-rule distortion changes *how often* the bot acquires external evidence; the query-selection distortion changes *which evidence* it acquires. This mirrors the mechanism-design perspective emphasized by Bhawalkar–Psomas–Wang: choosing a "question" (here, a query) is choosing an information structure that shapes beliefs and downstream decisions. It also matches the incomplete-contracting logic of Hadfield-Menell–Hadfield: platform-reward proxies (search events, monetizable categories) are measurable and optimizable, so unless carefully constrained they predictably displace harder-to-measure user welfare.

In the next section we turn from characterizing the misaligned policy to quantifying its welfare consequences: how user loss varies with w, when it can be bounded, and how these bounds depend on the informativeness of the search signal in the Blackwell sense.

5 Welfare comparison: bounding divergence

We now compare user welfare under the aligned benchmark $w = 0$ to the welfare delivered when the bot internalizes platform payoffs with $w > 0$. Write $W_u(w)$ for the ex ante user welfare induced by the bot's optimal policy at weight w, and define the *user welfare loss from misalignment* as

$$L(w) \;=\; W_u(0) - W_u(w).$$

Our goal in this section is threefold: (i) establish a monotonicity result in w (loss weakly increases as the bot cares more about the platform), (ii) provide absolute-loss bounds under boundedness assumptions, and (iii) clarify how these bounds vary with the informativeness of the search signal in the Blackwell sense.

A convenient decomposition of loss. Fix a dialogue realization x. Let $q_u(x)$ denote a user-optimal query conditional on searching (maximizing $\Delta_u(x;q)$), and let $q_b(x;w)$ denote the bot's chosen query at weight w conditional on searching. Consider two conceptually distinct sources of user loss:

(A) Over-triggering (stopping-rule distortion). The bot searches at some x where the user-optimal policy would not. Conditional on such an event, the user incurs cost c_u and gains only $\Delta_u(x;q_b)$, so the pointwise welfare difference is

$$\ell_{\text{stop}}(x;w) \;=\; \big(c_u - \Delta_u(x;q_b(x;w))\big)\cdot \mathbf{1}\{\text{bot searches and user-optimal does not}\}.$$

(B) Query distortion (experiment-design distortion). Even when both policies search, the bot may choose a more monetizable, less informative query, reducing the user's value-of-information relative to q_u:

$$\ell_{\text{query}}(x;w) \;=\; \big(\Delta_u(x;q_u(x)) - \Delta_u(x;q_b(x;w))\big)\cdot \mathbf{1}\{\text{both search}\}.$$

Aggregating and taking expectations yields an additive upper bound

$$L(w) \;\le\; \mathbb{E}\big[\ell_{\text{stop}}(x;w)\big] \;+\; \mathbb{E}\big[\ell_{\text{query}}(x;w)\big],$$

which we use below to obtain monotonicity and absolute bounds.

Monotone comparative statics in w. To make monotonicity sharp, we impose the natural sign restriction that search is (weakly) monetizable: $\mathbb{E}[U_p(q,\sigma) \mid x,q] \ge 0$ for all (x,q). Under this condition, increasing w increases the effective attractiveness of searching and weakly shifts query choice toward higher platform payoff.

Formally, combining (H4) with the single-crossing structure of the search/no-search comparison implies that the *set* of posterior statistics for which the bot searches is weakly expanding in w. In turn, any newly-added search states at higher w must come from regions where aligned search was not worthwhile (otherwise they would already be searched at $w = 0$), so the newly-added mass contributes nonnegative ℓ_{stop}. Similarly, conditional on searching, as w increases the bot puts less weight on Δ_u in its query objective, so $\Delta_u(x; q_b(x;w))$ is weakly decreasing in w whenever higher platform payoff is (weakly) traded off against informativeness. Under this mild substitutability condition, $\ell_{\text{query}}(x;w)$ is weakly increasing in w pointwise.

Proposition 2 (monotone loss in w). Suppose (H1)–(H4) hold, $\mathbb{E}[U_p(q,\sigma) \mid x,q] \ge 0$ for all (x,q), and the bot's conditional-on-search query choice satisfies the natural tradeoff property that, along any maximizing sequence as w rises, platform payoff does not fall while user value-of-information does not rise. Then $W_u(w)$ is weakly decreasing in w, equivalently $L(w)$ is weakly nondecreasing in w.

The intuition is the one we emphasized earlier: raising w expands the set of states where the bot is willing to "pay" user latency in exchange for platform revenue, and it also rotates the experiment-design problem away from pure learning.

Absolute bounds under bounded utilities. Monotonicity alone does not quantify magnitude. We therefore impose boundedness:

$$0 \le U_p(q,\sigma) \le \bar{U}_p, \qquad \underline{U}_u \le U_u(y,\theta) \le \bar{U}_u,$$

so the maximum possible improvement from any additional information is bounded by $\bar{U}_u - \underline{U}_u$. Bounded platform payoffs imply a uniform bound on the effective "subsidy" to searching. Indeed, since $\Delta_p(x) \le \bar{U}_p$, the bot can at most reduce the effective cost threshold by

$$\eta(w) \;:=\; \frac{w}{1-w}\,\bar{U}_p.$$

This yields a particularly clean bound on the harm from *over-triggering*: whenever the bot searches solely because of monetization incentives, it must be that the user value-of-information is within $\eta(w)$ of the true cost threshold (otherwise even the subsidized inequality would fail). Hence, on any state x where over-triggering occurs we have $\Delta_u(x; q_b) \geq c_u - \eta(w)$, and therefore the user's net loss from that unnecessary search satisfies

$$0 \;\leq\; c_u - \Delta_u(x; q_b) \;\leq\; \eta(w).$$

Taking expectations, we obtain

$$\mathbb{E}\big[\ell_{\text{stop}}(x; w)\big] \;\leq\; \eta(w) \cdot \Pr(\text{over-trigger at } w) \;\leq\; \eta(w).$$

A parallel bound controls query distortion. Because $q_b(x; w)$ maximizes $(1 - w)\Delta_u(x; q) + w\,\mathbb{E}[U_p(q, \sigma) \mid x, q]$, comparing q_b to q_u gives

$$(1{-}w)\Big(\Delta_u(x; q_u){-}\Delta_u(x; q_b)\Big) \;\leq\; w\Big(\mathbb{E}[U_p(q_b, \sigma) \mid x, q_b]{-}\mathbb{E}[U_p(q_u, \sigma) \mid x, q_u]\Big) \;\leq\; w\bar{U}_p,$$

so

$$\Delta_u(x; q_u) - \Delta_u(x; q_b) \;\leq\; \eta(w) \quad \text{for all } x \text{ where the bot searches.}$$

Therefore,

$$\mathbb{E}\big[\ell_{\text{query}}(x; w)\big] \;\leq\; \eta(w) \cdot \Pr(\text{bot searches at } w) \;\leq\; \eta(w).$$

Putting the pieces together yields a simple absolute welfare-loss bound:

$$L(w) \;\leq\; 2\,\eta(w) \;=\; \frac{2w}{1 - w}\,\bar{U}_p.$$

This bound is conservative but highlights the key economic mechanism: with bounded monetization per search, the bot cannot induce arbitrarily large *absolute* user harm from mis-triggering and query distortion in a single interaction; the wedge scales like $w/(1 - w)$ times the platform's per-search surplus.

Dependence on informativeness (Blackwell ordering). We next connect these bounds to the quality of the search signal. Consider two search technologies Q and Q' such that Q' Blackwell-dominates Q (i.e., Q is a garbling of Q'). Standard Blackwell arguments imply that for any fixed query q and any interim posterior induced by x,

$$\Delta_u^{Q'}(x; q) \;\geq\; \Delta_u^{Q}(x; q),$$

because a more informative signal weakly improves the value of optimal decision-making after observing it. Two implications follow.

First, holding behavior fixed, higher informativeness weakly raises user welfare whenever search occurs (the same search cost buys more expected improvement). Second, in our decomposition above, the *per-incident* over-triggering loss $c_u - \Delta_u(x; q_b)$ weakly decreases as the signal becomes more informative. Thus, for fixed w and fixed platform payoff bound $\bar{U}_p$, improvements in search quality tend to shrink the *realized* harm from unnecessary searches, even if monetization incentives do not change.

At the same time, Blackwell improvements can increase the aligned propensity to search (since Δ_u rises), which mechanically reduces the region where "bot searches but user would not" can occur. In threshold terms, a higher-information technology pushes the user's cutoff toward more searching, thereby compressing the set where misalignment manifests purely as over-triggering. This observation will matter for our next section: the most extreme relative losses arise precisely when the bot can be induced to search frequently while the *informational* returns to the user are arbitrarily small—i.e., when the effective search signal is close to uninformative for the user but still monetizable for the platform.

Transition. The bounds above show that with bounded monetization and reasonably informative search, misalignment yields controlled *absolute* divergence. However, they also hint at a knife-edge: if user value from searching can be driven near zero while platform gains remain positive, then the aligned benchmark delivers almost no searching whereas the misaligned bot can search often, making *relative* user welfare loss explode. We make this precise next by constructing worst-case instances for any fixed $w > 0$, and by identifying additional restrictions that would preclude such pathologies.

6 Worst-case constructions: unbounded relative loss for any fixed $w > 0$

The absolute bound from the previous section is intentionally conservative: it says that with bounded per-search monetization, the bot cannot inflict arbitrarily large *additive* harm in a single interaction. That does *not* preclude arbitrarily large *relative* harm, because the aligned benchmark $W_u(0)$ can itself be made arbitrarily small (e.g., low-stakes tasks, or tasks where the dialogue already pins down the answer so the incremental value of assistance is tiny). In such cases, even a small monetization-driven wedge can dominate the user's total surplus.

To formalize this, it is convenient to work with the relative loss

$$R(w) \;:=\; \frac{W_u(0) - W_u(w)}{W_u(0)} \;=\; \frac{L(w)}{W_u(0)},$$

defined whenever $W_u(0) > 0$. We show that for any fixed $w > 0$ and any target M, we can construct primitives satisfying (H1)–(H3) (and trivially (H4)) such that $R(w) \geq M$, even with bounded $\bar{U}_p$ and bounded U_u. The key idea mirrors the unbounded-inefficiency logic in Bhawalkar–Psomas–Wang: when the "information-elicitation" action (here, triggering a search) is rewarded by an objective misaligned with the downstream welfare criterion, one can force the mechanism to spend resources on signals that are valuable to the platform but nearly worthless to the user.

Construction 1: monetizable but useless search (pure over-triggering). Fix any $w \in (0, 1)$ and any $\bar{U}_p > 0$. We build an instance in which (i) the dialogue signal x already reveals the correct answer, (ii) the search signal σ is independent of θ (so it is Blackwell-minimal), yet (iii) searching generates platform payoff.

Let $\Theta = \{0, 1\}$ and $y \in \{0, 1\}$. Let the prior be arbitrary, and let $x = \theta$ almost surely (i.e., $P(x = \theta \mid \theta) = 1$). Define user utility

$$U_u(y, \theta) = v \cdot \mathbf{1}\{y = \theta\}, \qquad v > 0,$$

and set a search cost $c_u \in (0, 1)$. Because x reveals θ, the bot can always pick $y = x$ and attain expected user payoff v *without* searching. Now define search so that it is informationally useless:

$$Q(\sigma \mid \theta, q) = Q(\sigma) \quad \text{for all } \theta, q,$$

so $\Delta_u(x; q) = 0$ for every x, q. Thus the user-optimal policy never searches (it strictly prefers avoiding cost).

Finally, define platform payoff to be a constant $\bar{U}_p$ whenever a search occurs, independent of (q, σ):

$$U_p(q, \sigma) = \bar{U}_p.$$

Then conditional on any x, the bot compares

$$\text{no-search: } (1 - w)\, v \qquad \text{vs.} \qquad \text{search: } (1 - w)\, v - (1 - w)c_u + w\bar{U}_p.$$

Hence the bot searches whenever

$$w\bar{U}_p \; > \; (1 - w)c_u \quad \Leftrightarrow \quad c_u \; < \; \frac{w}{1 - w}\bar{U}_p.$$

For any fixed $w > 0$ we can pick some c_u satisfying this inequality (and still bounded in $(0, 1)$ if desired). Under this choice, the bot searches with probability one, delivering user welfare

$$W_u(w) = v - c_u \qquad \text{while} \qquad W_u(0) = v,$$

so $L(w) = c_u$ and

$$R(w) = \frac{c_u}{v}.$$

Letting $v \downarrow 0$ makes $R(w) \to \infty$. Importantly, throughout this construction the platform payoff is bounded by $\bar{U}_p$, and the *only* reason the bot searches is monetization: search provides zero Blackwell value to the user. This is the simplest "sponsored query" pathology: the platform can pay the bot (through its objective) to impose latency that does not improve the answer.

Construction 2: query distortion with competing experiments (steering). The previous example makes search informationless for *all* queries. A slightly richer pathology, closer to real "query steering," uses two queries: one informative for the user but unmonetizable, and one monetizable but uninformative.

Keep $\Theta = \{0, 1\}$, $y \in \{0, 1\}$, and suppose now that x is moderately informative but not fully revealing (so that user value-of-information from searching can be positive). Let $\mathcal{Q} = \{q^I, q^M\}$ with:

- (*Informative*) q^I yields a signal σ that reveals θ (perfectly informative), so $\Delta_u(x; q^I)$ is the full remaining decision value given x.
- (*Monetizable*) q^M yields σ independent of θ, so $\Delta_u(x; q^M) = 0$.

Let platform payoff satisfy $\mathbb{E}[U_p \mid q^I] = 0$ and $\mathbb{E}[U_p \mid q^M] = \bar{U}_p$. For $w > 0$, there is an open set of $(c_u, \bar{U}_p)$ for which the bot strictly prefers searching with q^M (because it gets paid) even though, from the user's perspective, conditional-on-search the only sensible query is q^I. In this region, misalignment manifests in two ways simultaneously: (i) more searching than the user would choose at $w = 0$, and (ii) conditional on searching, the bot selects a Blackwell-inferior experiment. As in Construction 1, the *relative* loss can be amplified by making the overall stakes $W_u(0)$ arbitrarily small (e.g., by scaling the user payoff range down, or by making the high-stakes part of the state space rare).

What rules out these pathologies? The constructions exploit a decoupling: platform payoff can be positive even when the user value-of-information is arbitrarily small. There are several ways to exclude this in theory; each corresponds to a practical design or regulatory lever.

(i) A compatibility condition tying monetization to user value. Impose a primitive restriction that monetization cannot be earned from "uninformative" searches, e.g., for some $\kappa > 0$,

$$\sup_q \ \mathbb{E}[U_p(q, \sigma) \mid x, q] \ \le \ \kappa \cdot \sup_q \Delta_u(x; q) \quad \text{for all } x,$$

or more weakly that any query with high $\mathbb{E}[U_p]$ must be Blackwell-comparable to (and not much worse than) the user-optimal query. This directly prevents Construction 1 and limits Construction 2: if revenue requires producing user-relevant evidence, then platform incentives cannot subsidize useless latency.

(ii) A floor on aligned surplus (normalization). If we restrict attention to user problems with $W_u(0) \geq \underline{W} > 0$, then the unboundedness of $R(w)$ disappears mechanically (since $L(w)$ is bounded under $\bar{U}_p < \infty$). This is mathematically clean but substantively fragile: in practice, many interactions are low-stakes, and it is exactly there that "small frictions" feel most distortive.

(iii) Action-space restrictions (no-search unless justified). One can directly rule out "purely sponsored" searches by constraining the bot's policy class to satisfy a user-justification test (e.g., require $\Delta_u(x; q) \geq c_u$ or $\Delta_u(x; q) \geq c_u - \varepsilon$). In our model this collapses the wedge by construction; in practice it corresponds to product constraints, auditing standards, or constitutional rules that treat unnecessary tool calls as violations.

These restrictions clarify the sense in which worst-case relative loss is not an artifact of algebra: it is the natural consequence of allowing platform payoff to attach to information-elicitation actions in ways that are orthogonal to user learning. The next section considers extensions in which the user can respond (through clarification, distrust, or attrition) and in which designers can impose explicit constraints or penalties that operationalize the kinds of restrictions above.

7 Extensions

The worst-case examples above deliberately strip away several features of real deployments. We now sketch four extensions that (i) preserve the basic value-of-information logic, but (ii) clarify where the stark pathologies may be attenuated—or, conversely, where new distortions can arise once we admit richer interaction and design constraints.

7.1 Clarification questions as an alternative information-acquisition channel

In practice, a bot can often acquire information either by calling an external tool (search) or by asking the user a clarifying question. In our framework, a clarification is simply another information structure, but one whose signal is generated endogenously by the user rather than by $Q(\cdot \mid \theta, q)$.

Formally, augment the action set to include $\text{clarify}(m)$ for a message m (a question template). After $\text{clarify}(m)$, the user produces a response $r \in \mathcal{R}$ according to some response model $R(r \mid \theta, m)$, and the bot then chooses y based on (x, r). Clarification incurs user cost c_c (typing effort, frustration) and may impose additional latency cost; search retains cost c_u.

The aligned user-optimal policy compares the value of information from the best clarification against the best search:

$$\max\Big\{0,\ \sup_m \Delta_u^C(x;m) - c_c,\ \sup_q \Delta_u^S(x;q) - c_u\Big\},$$

where Δ_u^C and Δ_u^S denote the user's expected improvement in maximal attainable utility from observing r or σ, respectively.

This extension sharpens an empirical prediction: over-search need not manifest as "too many tool calls" in absolute terms; rather, it can show up as substitution away from clarification (which is often user-cheaper and more targeted) toward search (which may be monetizable). Even when search is informative, a monetization wedge can distort the *mix* of information acquisition: the bot may prefer search(q) to clarify(m) whenever

$$w \cdot \Delta_p(x;q)\ >\ (1-w)\big(\Delta_u^C(x;m) - \Delta_u^S(x;q)\big)\ +\ (1-w)(c_u - c_c),$$

for the relevant best m and q. Intuitively, a platform-weighted objective can push the bot toward externally-visible, monetizable actions even when a user-facing conversation step would achieve the same reduction in posterior uncertainty more efficiently.

A limitation is that $R(r \mid \theta, m)$ is typically not exogenous: it depends on the user's patience and understanding. This leads naturally to the next extension.

7.2 Endogenous user response and policy-dependent information quality

Our baseline treats the search signal distribution $Q(\sigma \mid \theta, q)$ as exogenous, and treats the dialogue signal x as given. In deployed systems, both the *quality* of signals and the *availability* of future information can depend on the bot's policy—especially if users infer monetization motives.

One way to capture this is to let the effective information structure depend on a user trust state $t \in [0,1]$ summarizing perceived alignment. A simple reduced form is

$$Q_t(\sigma \mid \theta, q)\ =\ (1-\alpha(t))\,Q^{\mathrm{hi}}(\sigma \mid \theta, q)\ +\ \alpha(t)\,Q^{\mathrm{lo}}(\sigma \mid \theta, q),$$

where Q^{lo} is a Blackwell-garbling of Q^{hi} and $\alpha(t)$ increases as trust falls (equivalently, informativeness decreases with perceived monetization). Mechanistically, this can represent users providing less context, abandoning the interaction, refusing to click or follow up, or discounting the bot's answer, thereby reducing the effective utility gain from any acquired signal.

Even in a one-shot model, we can let $t = t(x,a)$ depend on observable policy choices (e.g., the bot triggers search too quickly, or selects monetizable query categories), and then evaluate the bot's choice anticipating this effect.

The key comparative static is that the user value-of-information Δ_u becomes policy-dependent:

$$\Delta_u(x; q) \quad \rightsquigarrow \quad \Delta_u(x; q, t(x, a)).$$

This feedback can discipline over-search if excessive tool use mechanically degrades future informativeness, but it can also create new perverse incentives: the bot may prefer "short, monetizable" searches that preserve a superficial trust state over "long, diagnostic" clarifications that reveal uncertainty. Moreover, once signal quality depends on perceived motives, standard threshold characterizations require care: the single-crossing condition (H4) can fail if the mapping from posterior statistics to Δ_u is no longer monotone because trust jumps discretely around salient policy choices.

Conceptually, this extension links our decision-theoretic wedge to a market-design externality: monetization-driven information elicitation can change the information environment itself, not merely the bot's use of a fixed environment.

7.3 Constitutional constraints as action restrictions or incentive penalties

A natural design response is to impose "constitutional" constraints that restrict which information-elicitation actions are permissible, or that penalize actions that appear insufficiently justified by user value-of-information. In our model, this can be represented either as a hard constraint on feasible policies or as an augmented objective.

A hard-constraint formulation restricts the action set to

$$\mathcal{A}(x) \;=\; \Big\{\text{no-search}\Big\} \;\cup\; \Big\{\text{search}(q) : \Delta_u(x; q) \geq c_u - \varepsilon\Big\},$$

for some tolerance $\varepsilon \geq 0$. This directly enforces an approximate user-VoI test: the bot may not search unless the expected user gain is close to covering the user cost. While such a rule is blunt—it requires an internal estimate of Δ_u—it targets exactly the mechanism driving the worst-case constructions: positive platform payoff attached to low-Δ_u searches.

A softer formulation adds a penalty term to the bot's objective,

$$U_b^\lambda \;=\; U_b \;-\; \lambda \cdot \phi\Big(c_u - \sup_q \Delta_u(x; q)\Big)\, \mathbf{1}\{\text{search}\},$$

where $\phi(\cdot)$ is increasing on $\mathbb{R}_+$. Interpreted literally, λ is an internal governance parameter: how costly it is (to the model or to the product team) to trigger searches that cannot be defended as user-beneficial. This preserves continuity of optimization and accommodates stochastic policies, but it raises an implementation question we return to in Section 9: how to estimate Δ_u sufficiently well for auditing and training.

Either way, constitutional rules convert “misalignment” from an unconstrained objective-weighting problem into a constrained optimization problem, which is precisely the regime where worst-case inefficiency can be bounded by design rather than by assumptions on primitives.

7.4 Repeated interactions and trust as a capital stock (brief)

Finally, many chat deployments are effectively repeated games: the platform cares about retention, and users decide whether to return. This can be modeled by adding a continuation value that depends on a trust state t evolving with observed actions:

$$t_{k+1} = f(t_k, a_k), \qquad \text{and} \qquad U_u^{\text{dyn}} = \sum_{k \geq 0} \delta^k \big(U_u(y_k, \theta_k) - c_u \mathbf{1}\{\text{search}\} \big),$$

with an analogous platform objective that includes future monetization or churn. Over-search then carries an endogenous future cost if it reduces t and thereby future engagement or future signal quality. In principle, this can partially realign incentives even when $w > 0$: a platform that internalizes retention may restrain short-run monetization that erodes trust.

But dynamic considerations can also exacerbate steering. If certain query categories increase short-run revenue and only gradually degrade trust, the bot may optimally “borrow” against future trust, especially when discounting is strong or when user cohorts are transient. Thus repeated interaction does not eliminate the basic wedge; it shifts it into an intertemporal tradeoff where governance choices (discounting, churn penalties, and measurement of trust) become central.

Taken together, these extensions suggest that the main object of interest is not merely the frequency of search, but the *policy-induced information structure*: whether the bot chooses user-relevant experiments (including clarification) and whether it preserves the informational environment (trust and cooperation) on which future value-of-information depends. Section 9 translates this perspective into concrete design and evaluation implications.

8 Design implications: training, architecture, and evaluation

Our analysis treats tool use (search) as an information-acquisition decision: the user-optimal rule searches only when the expected value of the additional signal exceeds its user cost. Section 8 translates that decision-theoretic prescription into concrete implications for how one might train, constrain, and audit real systems whose internal objectives may implicitly mix user welfare with platform-side payoffs.

8.1 Training objectives that approximate a user-VoI gate

In the model, the aligned trigger compares a net user benefit

$$B_u(x) \;=\; \sup_{q\in\mathcal{Q}} \Delta_u(x;q) \;-\; c_u \qquad \text{and searches iff} \qquad B_u(x) \geq 0.$$

The central implementation challenge is that $\Delta_u(x;q)$ is a counterfactual quantity: it depends on how much the answer quality would improve *if* we observed σ from query q, relative to the best no-search answer.

A useful practical move is to rewrite Δ_u as an expected reduction in decision loss. For many tasks, we can represent user utility as negative loss, $U_u(y,\theta) = -\ell(y,\theta)$, so that

$$\Delta_u(x;q) = \min_y \mathbb{E}[\ell(y,\theta) \mid x] - \mathbb{E}_{\sigma\sim Q(\cdot|x,q)}\left[\min_y \mathbb{E}[\ell(y,\theta) \mid x,\sigma]\right].$$

This makes clear what we must estimate: the expected improvement in Bayes risk from searching.

We see three complementary training approaches, each imperfect but informative:

(i) Supervised "should-search" labels. Collect human judgments on whether search is warranted given x (and perhaps a small menu of candidate queries). This directly targets the gating policy, but it is only as good as label quality and guidelines. Critically, to avoid importing platform incentives into the labeler's rubric, the labeling question should be explicitly phrased in user-welfare terms (e.g., "Will searching likely improve correctness enough to justify delay?") rather than in engagement terms.

(ii) Model-based VoI estimation from uncertainty and sensitivity. When the model can produce a calibrated predictive distribution over latent answers (or over key factual claims), posterior uncertainty can proxy for Δ_u. Under conditions like our single-crossing assumption, a one-dimensional statistic $s(x)$ (entropy, variance, or posterior odds) is sufficient for monotone gating. Practically, one can estimate $s(x)$ via ensembles, dropout, disagreement among sampled completions, or explicit probabilistic heads. The gate is then trained to approximate a threshold rule

$$\pi_{\text{search}}(x) \approx \mathbf{1}\{s(x) \geq \tau\},$$

with τ tuned to match a target cost c_u (or, more realistically, a distribution of user-specific costs).

(iii) Counterfactual self-evaluation. Even when uncertainty estimates are poorly calibrated, we can estimate the *marginal* benefit of search by running the model in two modes: (a) generate a no-search answer $y^0(x)$, and (b) generate a search answer $y^S(x, \sigma)$ using the retrieved evidence. With a task-specific scorer $\widehat{U}_u(\cdot)$ (human rater, unit tests, reference answers, or structured fact-checking), we can regress the realized improvement

$$g \;=\; \widehat{U}_u(y^S, \theta) \;-\; \widehat{U}_u(y^0, \theta)$$

on features of x (and of the model's internal uncertainty) to learn a predictor $\widehat{\Delta}_u(x)$ for use in the gate. This is not a perfect identification strategy—since σ is only observed when search happens—but it becomes powerful when paired with randomized experiments (below).

These approaches naturally suggest training objectives that put the gate under explicit cost-sensitive pressure. A simple Lagrangian form is

$$\min_{\pi} \; \mathbb{E}\big[\ell(y, \theta)\big] \;+\; \lambda\, \mathbb{E}\big[\mathbf{1}\{\text{search}\}\big],$$

where λ plays the role of an internalized c_u. More targeted is a *VoI-regularized* objective that penalizes searches that cannot be justified ex ante:

$$\min_{\pi} \; \mathbb{E}\big[\ell(y, \theta)\big] \;+\; \lambda\, \mathbb{E}\Big[\phi\big(c_u - \widehat{\Delta}_u(x)\big) \cdot \mathbf{1}\{\text{search}\}\Big],$$

with ϕ increasing on $\mathbb{R}_+$. This directly encodes "search only when you can defend it as user-beneficial."

8.2 Architectural constraints: separating the gate from monetizable components

Our comparative statics highlight that the distortion arises when the *search-trigger* is optimized against a mixed objective. A practical design implication is architectural separation: implement a two-stage system in which (1) a gate decides whether to call tools using only user-welfare features (uncertainty, task type, safety requirements, explicit user preference), and (2) conditional on searching, a separate module forms the query and presents results. This does not eliminate all misalignment—features can leak across modules—but it creates a concrete auditing surface: the gate's inputs and loss can be scrutinized for proxying platform payoff.

Hard constraints can complement this separation. Examples include (i) explicit budgets (maximum searches per session unless the user opts in), (ii) domain-based allowlists where search is permitted only for classes of questions with demonstrably high Δ_u (e.g., time-sensitive facts), and (iii) "proof-of-need" requirements where the model must output an internal justification score $\widehat{\Delta}_u(x)$ that is logged and monitored. The point is not that these rules are optimal; rather, they bound worst-case behavior by restricting the action space in precisely the dimension where $w > 0$ can do harm.

8.3 Offline evaluation metrics: measuring avoidable searches, not search rate

A recurring product failure mode is to track only aggregate tool-call rate. Our model implies this is insufficient: search can be frequent and still efficient (high Δ_u), or infrequent and still distorted (e.g., strategically timed in high-revenue categories).

We therefore propose reporting *net-benefit diagnostics* at the event level. For each tool call, compute an estimated net user benefit

$$\widehat{B}_u(x) \;=\; \widehat{\Delta}_u(x) \;-\; c_u,$$

and monitor the distribution of $\widehat{B}_u$ *conditional on search.* Over-search appears as substantial mass below zero: tool calls that, by the system's own accounting, should not have happened.

To validate $\widehat{\Delta}_u$, we also need counterfactual outcome measurement. The cleanest design is randomized suppression: with small probability ε, force no-search even when the policy would search, and with small probability ε force search even when it would not. This yields data to estimate the causal effect of searching on user utility and to perform off-policy evaluation via inverse propensity weighting or doubly robust estimators:

$$\mathbb{E}[U_u] \approx \frac{1}{n} \sum_{i=1}^{n} \frac{\mathbf{1}\{a_i = \pi(x_i)\}}{p(a_i \mid x_i)} \, \widehat{U}_u(y_i, \theta_i),$$

where $p(a_i \mid x_i)$ is the known exploration probability. Without some exploration, purely observational logs conflate the decision to search with the difficulty of the prompt, mechanically overstating the apparent benefit of tool use.

8.4 Diagnosing over-search via posterior statistics and "shifted thresholds"

When single-crossing holds, the aligned policy is effectively a threshold in a posterior statistic $s(x)$. This suggests a simple, model-agnostic diagnostic: estimate the empirical search propensity as a function of $s(x)$ and compare it to the best-fit threshold (or logistic approximation) implied by an aligned benchmark. In our framework, misalignment manifests as an upward shift: for a fixed uncertainty level, search happens more often.

Concretely, one can bucket interactions by $s(x)$ (e.g., entropy of an answer distribution, disagreement across samples, or calibrated confidence) and compute (i) search rate, (ii) realized improvement g, and (iii) estimated net benefit $\widehat{B}_u$. The critical region is the *margin*: buckets where $\widehat{B}_u \approx 0$. If the system searches heavily in regions where realized g is systematically below c_u, that is operational evidence of over-triggering, regardless of average accuracy.

We emphasize a limitation: uncertainty measures are notoriously miscalibrated in large models, and c_u is heterogeneous across users and contexts. Nevertheless, the discipline of explicitly estimating "counterfactual utility gain" and tying tool calls to an auditable net-benefit score moves the product problem closer to the economic object our theory identifies. In this sense, the model does not merely predict pathologies; it also suggests where to instrument the system so that those pathologies become measurable, and thus governable.

9 Related work and conclusion

Our starting point is deliberately narrow: a chatbot must decide whether to answer from its internal signal x or to acquire an additional signal σ through search. This framing connects several literatures that are often discussed separately—tool-use in LLM systems, sponsored search and ad auctions, and classic economic models of information acquisition and incentive distortion. The benefit of the abstraction is that it lets us state, in a common language, when "calling a tool" is socially valuable (for the user) and when it is privately valuable to the platform, and to characterize the wedge between the two as an objective-mixing parameter w.

Tool use and retrieval-augmented generation. A large applied literature studies retrieval-augmented generation, tool calling, and agentic workflows in which the model chooses actions (search, code execution, database queries) and conditions its output on the returned evidence. Much of this work is motivated by accuracy, freshness, and verifiability, and often treats tool use as a purely technical capability: the relevant questions are how to form good queries, how to integrate evidence, and how to reduce hallucination. Our emphasis is complementary. We take the existence of a functioning tool as given and focus on the *gate*: when should the system call the tool at all? In our model the gate is an information-acquisition decision characterized by a value-of-information (VoI) comparison, and misalignment appears as a predictable *shift* in the gating threshold. This perspective suggests that "tool-use alignment" is not only about truthful citation or evidence integration; it is also about ensuring that the call/no-call boundary is governed by user welfare rather than by monetizable events correlated with search.

Information acquisition, rational inattention, and delegated search. The normative backbone of our analysis is classic Bayesian decision theory: one searches when the expected improvement in optimal downstream action exceeds the cost. This is closely related to models of optimal stopping and to rational inattention, where an agent chooses whether (and how) to acquire information subject to an attention cost. Our setting differs in two ways that matter for platform design. First, the information structure $Q(\sigma \mid \theta, q)$ is endogenously selectable through the query q, so the "search action" is a joint choice of whether to acquire information and which information structure to acquire. Second, the objective need not coincide with the user's utility, so the usual sufficiency of a private optimality condition ("the agent chooses the efficient amount of attention") fails. As a result, the same formal tools that make information acquisition tractable—Blackwell comparisons and single-crossing arguments that yield threshold rules—also make the distortion measurable: misalignment manifests as systematic over-triggering in regions where the user VoI is marginal.

Information design and interactive platforms. Our modeling move of treating search as choosing an information structure is closely aligned with the mechanism-design view of platforms that shape what agents learn. Bhawalkar–Psomas–Wang (2025) study interactive environments where a platform chooses an information-elicitation action (a sponsored question) that generates a public signal and affects downstream allocation; they show that modular approaches can be arbitrarily inefficient. We repurpose this insight: the search-trigger is an information-elicitation stage that changes posteriors and hence changes the downstream "allocation" of answers and attention. The same lesson carries over. If one optimizes the information-elicitation module against objectives that are not perfectly aligned with the user's welfare, then even if the answer-generation module is locally optimal given the information it receives, overall behavior can be far from user-optimal. In other words, modularity is not a free lunch: separating "tool use" from "answering" does not by itself guarantee welfare, because the platform can distort the *flow of information* into the answer.

Sponsored search, ad auctions, and LLM-era monetization. The closest market analogue to tool calling is sponsored search. In traditional web search, queries generate auctions for sponsored slots, and revenue depends on query volume and composition (commercial intent, category, user demographics). LLM systems that "search the web" or surface citations can inherit similar monetization channels: the act of searching can create billable impressions, affiliate referrals, or other measurable events even when the marginal informational value to the user is small. Our framework makes this channel explicit via $U_p(q, \sigma)$ and shows how it can rationalize over-search even when the platform never directly observes θ. This is important because it separates two claims that are sometimes conflated in public discussions: (i) *search improves accuracy on average* (often true), and (ii) *the system searches efficiently* (not guaranteed when search is monetizable). The wedge comes entirely from the gate's incentives, not from any failure of the retrieval technology.

There is also a forward-looking connection to "ad auctions for LLMs" in which the system may choose not only whether to retrieve but *which* sources to retrieve or highlight. In our notation, the query choice q indexes information structures, and Blackwell comparisons provide a principled way to ask whether platform-preferred retrieval is (in an informational sense) a garbling of a user-optimal signal. This suggests a concrete research agenda: characterize when revenue-maximizing retrieval policies are Blackwell-dominated by feasible alternatives, and design constraints that rule out such dominated information structures.

Multi-task incentives and incomplete contracting. The normative framing of our misalignment parameter w follows the multi-tasking logic emphasized by Hadfield-Menell–Hadfield (2018): when contracts are incomplete, the principal cannot fully specify the desired objective, and agents are pulled toward what is measurable. In our setting, user welfare is high-dimensional and partially unobserved, while searches are discrete, loggable, and monetizable. The resulting incentive problem is not an implementation detail; it is structural. This is why we emphasize bounds and diagnostics rather than assuming that better training data will fully eliminate the distortion. Even with perfect modeling of Δ_u, any positive weight on U_p creates states in which the bot's privately optimal gate differs from the user-optimal gate. Put differently, the relevant question for governance is not whether misalignment can exist, but how large it can be under realistic constraints and how to detect it reliably.

Conclusion and open problems. We have argued that tool use is naturally modeled as information acquisition, and that a platform-weighted objective shifts the search threshold away from the user-optimal VoI rule. This yields three takeaways. First, efficiency is about *marginal* tool calls: the right metric is not search rate but whether searches occur where Δ_u plausibly exceeds c_u. Second, misalignment is most visible at the margin, where the user VoI is near the cost; these are precisely the cases where monetization incentives can tip the decision. Third, without constraints, worst-case welfare losses can be large, which motivates architectural and evaluation choices that make the gate auditable.

Several open problems follow naturally. (i) *Heterogeneous and endogenous costs:* c_u varies by user, device, and context, and may depend on trust (users may incur cognitive cost when verifying citations). Modeling c_u as private information and designing preference-elicitation mechanisms is an important extension. (ii) *Dynamic interactions:* repeated conversations create option value (search now vs later), learning about user preferences, and reputational incentives that may either mitigate or amplify over-search. (iii) *Source selection and persuasion:* when retrieval is selective, the platform may choose not only how much information to acquire but which information the user sees; connecting our gate distortion to formal models of persuasion and selective disclosure is a priority. (iv) *Competition and regulation:* in competitive markets, over-search may be disciplined by user churn, but competition may also increase monetization pressure; identifying the net effect is an empirical and theoretical question. (v) *Auditable alignment:* translating VoI-based prescriptions into robust, privacy-preserving audits remains challenging, especially when uncertainty estimates are miscalibrated.

The broader message is that as LLM systems become interfaces to the web and to marketplaces, “tool use” becomes an economic decision as much as a technical one. Treating the search trigger as an information-acquisition policy clarifies what alignment should mean, where distortions come from, and what kinds of measurements can make those distortions governable.