End-to-End vs.˜Modular Training for Agentic Search: A Price-of-Anarchy Theory for Chatbot Tool Use

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January 6, 2026

Abstract

We study agentic chatbots that decide whether to trigger web search and how to use the resulting information before producing a final answer. The decision to search is an information-acquisition choice: it can improve answer quality by revealing a signal about the user's latent intent, but it incurs latency/cognitive costs and may generate platform-side benefits (e.g., monetizable search events). Motivated by the growing deployment of conversational assistants and by recent mechanism-design results showing severe inefficiency under modularized information acquisition *versus* allocation **?**, we analyze an analogous modularity gap arising from *training architecture* rather than auction rules. We formalize a broad class of modular pipelines in which a stage-1 component chooses whether to search using a proxy objective (such as predicted engagement or revenue), while a stage-2 component optimizes answer quality conditional on the realized search signal. Our main theorem shows an unbounded Price of Anarchy: for any constant C, there exist environments where every such modular pipeline achieves user welfare at least a factor C worse than the end-to-end Bayes-optimal policy, even if the stage-2 component is pointwise optimal given the stage-1 choice. We then provide sufficient conditions under which modularity becomes safe: if the stage-1 proxy is a certifiable approximation to the *user value-of-information* of searching and the stage-2 model is calibrated, then the welfare loss is uniformly bounded. Conceptually, our results connect economic theories of incomplete contracting and reward misspecification **?** to practical LLM systems: optimizing easily measurable proxies for tool use can lead to arbitrarily harmful distortions unless training, architecture, or audits are designed to internalize downstream user welfare.

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tainty with misaligned incentives. Explain why modular training is attractive in practice and why it can fail. Summarize contributions: (i) formal model and welfare benchmark, (ii) unbounded PoA for modular training, (iii) bounded-loss conditions via value-of-information certificates. Position relative to interactive sponsored-question auctions and incomplete contracting.

2. 2. **Model.** Specify latent intent $\theta \sim D$, dialogue observation x, tool-use action s (search/no-search; optional query q), signal generation $\sigma \sim Q_{s,q}(\cdot \mid \theta)$, and final answer y. Define user welfare W and (optionally) platform benefit B. Clarify information structure (what is observed by whom) and measurability assumptions. Discuss extensions: multiple search calls; clarifying questions as additional actions.

3. 3. **Benchmarks: End-to-End Optimality and Value of Information.** Define the end-to-end user-optimal policy $\pi^\star$ as Bayes-optimal. Derive the canonical value-of-information threshold structure for searching under regularity conditions. Define $\Delta\text{VoI}(x)$ and show how the optimal search decision depends on comparing $\Delta\text{VoI}(x)$ to the search cost c.

4. 4. **A Class of Modular Training Pipelines.** Formalize modularity as two separate maximizations: stage-1 chooses (s, q) to maximize a proxy J_1 and stage-2 chooses y to maximize J_2 given realized (x, s, q, σ). Map common engineering choices to this abstraction (separate classifier for tool use; retrieval-augmented generation module; reward models that include search events). Provide at least two concrete instantiations: (i) monetization-weighted proxy, (ii) engagement proxy.

5. 5. **Main Negative Result: Unbounded Price of Anarchy.** State and prove the unbounded PoA theorem. Construct an explicit family of instances where searching is socially valuable (raises user welfare) but reduces the stage-1 proxy (e.g., because proxy rewards search regardless of helpfulness, or because proxy values outcomes that are negatively correlated with informative searches). Show that even with an optimal stage-2 policy, stage-1 proxy mis-selects the information structure. Provide variations: over-search case and under-search case; query-steering variant where q is distorted.

6. 6. **Positive Results: When Modularity is Safe.** Give sufficient conditions under which modular pipelines approximate end-to-end optimality. Introduce the notion of a *value-of-information certificate* proxy: J_1 estimates the incremental user value of search (or a lower bound). Prove additive and multiplicative welfare guarantees as functions of proxy error (ϵ, δ), bounded utility range, and (optional) Lips-

chitz continuity. Discuss the role of calibration/proper scoring rules in stage-2 so that J_2 corresponds to posterior expected utility.

7. 7. **Design Implications for Training and Architecture.** Translate the theory into system design choices: (i) end-to-end evaluation for tool-use, (ii) constrained optimization / regularization that ties stage-1 reward to estimated ΔVoI, (iii) post-hoc auditing of search justification, (iv) architectural constraints limiting query steering. Provide a short taxonomy: what fixes the negative example and what does not.

8. 8. **Empirical / Measurement Plan (Optional but Recommended).** Propose an operationalization of ΔVoI using counterfactual evaluation: measure answer quality with and without search via offline oracle labels or LLM-as-judge with calibrated uncertainty; estimate the distribution of unjustified searches. Suggest experiments varying proxy incentives and checking monotone changes predicted by theory (e.g., increasing monetization weight increases search frequency and decreases measured user welfare).

9. 9. **Related Work.** Discuss connections to: interactive ad mechanisms and unbounded PoA from modular auctions **?**; incomplete contracting and reward misspecification **?**; multi-task incentive distortions; RLHF/RLAIF and reward hacking; information acquisition and optimal stopping; algorithmic auditing and compliance.

10. 10. **Conclusion and Open Problems.** Summarize contributions and list next steps: multi-round conversations, endogenous user response to steering, multiple principals with heterogeneous users, robust/strategic agents, and combining auditing with uncertainty over objectives.

1. Introduction and Motivation. Modern conversational assistants increasingly operate as *agentic* systems: they do not merely map a user message to an answer, but instead choose among intermediate actions such as calling external tools (e.g., web search, retrieval over proprietary corpora, code execution), selecting a query, and then composing a final response. Even in the seemingly simple case of a single search call, the assistant faces a sequential decision problem under uncertainty. The user's true intent—what facts are needed, what level of rigor is expected, what constraints matter (freshness, jurisdiction, personalization), and what counts as a satisfactory answer—is only partially revealed by the dialogue. The assistant must infer this latent intent from the observed conversation and decide whether the incremental benefit of acquiring outside information outweighs the associated costs.

This paper frames *search triggering and query phrasing* as an instance of information acquisition in the decision-theoretic sense. The key observation is that "using a tool" is not simply a means of producing text; it is a choice of an *information structure* that determines what signal the system will observe before committing to a final answer. In classical Bayesian decision theory, an agent compares the value of additional information to its cost, and optimally acquires information when the expected gain in decision quality exceeds the acquisition cost. When a chatbot decides whether to search, it is implicitly solving this value-of-information tradeoff: searching may reduce hallucinations and improve factuality, but it also introduces latency, interaction friction, potential privacy exposure, and possibly new failure modes (e.g., over-trusting low-quality sources or misreading retrieved snippets).

Why this is not just a modeling nicety. The practical relevance of this framing becomes clear once we recognize that tool use is often trained and deployed *modularly*. In production systems, a tool-use component (or policy head) may be optimized to predict whether a search call will yield engagement, satisfy platform policies, or generate monetizable events. Separately, a response generator is trained to produce a high-quality answer *conditional* on whatever information is available—either no external information, or a retrieved bundle of documents. This division is attractive: it reduces engineering complexity, allows different teams to own different modules, and provides clean supervised targets (e.g., "should we search?" labels inferred from logs). It also reflects real constraints: large language models are expensive to train end-to-end with interactive exploration, while a standalone trigger classifier can be iterated rapidly.

However, modularity introduces a subtle but central failure mode: even if the response generator is "perfect" given its inputs, the overall system can be arbitrarily far from user-optimal if the tool-use module selects the wrong

information structure. The system's welfare is determined not only by how well it answers given retrieved evidence, but by whether it *chose* to retrieve the right evidence in the first place. This point is easy to miss because evaluation practices often condition on tool calls ("given that search happened, did the model cite correctly?") rather than evaluating the upstream decision ("should it have searched at all?"). Yet the user experiences the composition of these decisions.

Misaligned incentives and proxy objectives. The incentive misalignment we study is not hypothetical. The user's objective is naturally expressed as expected answer quality minus the cost of tool use (latency, annoyance, privacy risk, cognitive overhead). The platform, by contrast, may optimize a proxy: search calls may increase session length, create measurable events, or support product goals that correlate only imperfectly with user welfare. Even if the platform's proxy is benign (e.g., "reduce hallucinations"), it remains a proxy because the true downstream utility is hard to observe and can be context-dependent. This tension echoes a broader alignment theme: optimizing what is easy to measure can distort what is hard to measure. Hadfield-Menell and Hadfield's incomplete-contracting perspective formalizes why such reward misspecification is routine rather than exceptional: when objectives are complex and partially unverifiable, systems are necessarily trained on incomplete proxies, and the resulting behavior can systematically deviate from the intended goal.

Our setting adds an additional layer: the proxy is often applied to the *information acquisition stage*, not merely to the final output. This matters because information acquisition changes the feasible set of downstream actions. A mis-optimized trigger policy can "lock in" an inferior information structure, after which even an optimal answerer cannot recover the lost welfare. In other words, the proxy does not merely bias a choice among answers; it biases the choice among *experiments* the system runs on the world.

A motivating example. Consider a user who asks: "Is the new tax credit available for used EVs in my state?" The latent intent includes jurisdiction, date, and eligibility details; the dialogue signal may not specify all of these. A user-optimal assistant would likely search (or ask a clarifying question) because the answer is time-sensitive and state-dependent. A proxy-trained trigger, however, might learn that users who receive a quick confident response are less likely to abandon the session, and thus prefer NoSearch in ambiguous cases. The downstream generator might be highly accurate when provided with relevant statutes, but it never sees them because the upstream module refused to search. Conversely, the proxy might over-trigger search for queries that are easy to answer from parametric knowledge, creating unnecessary latency and friction. Both errors are welfare losses; crucially, they

arise upstream of the "answer quality" module.

Connection to information acquisition and mechanism design. Our analysis is inspired by a conceptual parallel with interactive ad systems and sponsored-question auctions. In such systems, a platform may choose which questions to ask users (or which information to elicit) before allocating ad slots. Bhawalkar, Psomas, and Wang (2025) show that modularity between *information acquisition* and *allocation* can lead to unbounded inefficiency: selecting an information structure based on induced intermediate utilities can be arbitrarily worse than selecting information end-to-end for the final objective. We adapt the logic to conversational assistants. The "allocation" stage corresponds to generating the final answer once a signal is realized, while the "information acquisition" stage corresponds to deciding whether and how to search. The analog of their inefficiency is that a trigger policy optimized against a proxy can systematically pick low-welfare information structures, producing an unbounded price-of-anarchy-style gap relative to a fully user-optimal policy.

The broader message is not that modularity is always bad, but that modularity is a *structural constraint* that can interact with misalignment in a particularly harmful way. When the upstream module is trained on a proxy objective, downstream optimality cannot rescue the system because the relevant information was never acquired.

Our approach and contributions. We develop a decision-theoretic model that makes this issue precise and yields two types of results: an impossibility result that formalizes the potential severity of modular misalignment, and a positive result that identifies conditions under which modularity is safe.

First, we propose a simple welfare benchmark grounded in Bayesian decision theory. The benchmark is the end-to-end Bayes-optimal policy that jointly chooses (i) whether to search and how to phrase a query, and (ii) what final answer to return after observing the resulting signal. This benchmark captures the normative "search if and only if the incremental value-of-information exceeds the cost" principle, generalized to the setting where the assistant can design its own query and then condition its answer on what it observes. The model is intentionally minimalist: it abstracts away from the details of ranking algorithms and language generation in order to isolate the core economic tradeoff between information and cost.

Second, we formalize a broad class of modular training pipelines in which a stage-1 tool-use module chooses the search action using a proxy objective, while a stage-2 response module is (potentially) user-optimal conditional on the chosen action and observed signal. Within this class, we show that modularity can be arbitrarily inefficient: for any constant $C > 0$, there exist environments where the user-optimal end-to-end policy achieves at least

C times the welfare of the modular policy. Importantly, this gap persists even when the stage-2 answerer is *pointwise optimal* given its inputs. The inefficiency is thus not about a weak language model; it is about selecting the wrong experiment. Conceptually, this aligns with the unbounded inefficiency phenomena in modular information acquisition mechanisms: an upstream proxy can push the system toward systematically bad information structures, and downstream optimality cannot undo the loss.

Third, we provide a positive counterpart: modularity can be made safe if the stage-1 proxy is a *certifiable* approximation to the user's value-of-information for searching. Concretely, we define conditions under which the proxy objective tracks, up to small error, the incremental expected utility that search would provide relative to not searching. When such a certificate holds uniformly (in the relevant posterior states induced by dialogue), a modular trigger that maximizes the proxy will be near-optimal for user welfare, and the welfare loss admits a uniform bound. This result does not assume perfect reward specification—in line with the incomplete-contracting view—but instead asks for bounded divergence between the proxy and the true value-of-information. The practical takeaway is that we can tolerate modular training if we can *audit* the stage-1 objective against a decision-theoretic quantity (the value of information), rather than assuming the proxy is aligned by construction.

Why the value-of-information lens is useful in practice. The certificate viewpoint suggests a concrete path for system design: rather than labeling tool use from engagement logs or from heuristics alone, we can attempt to estimate the incremental benefit of search in terms of downstream answer quality, and train the trigger to predict that increment. This reframes "should we search?" as a prediction task about counterfactual improvement: how much better would the answer be if we searched? While estimating such counterfactuals is nontrivial, the point of our theory is to clarify *what* must be approximated to obtain welfare guarantees. In particular, it is not enough for a proxy to correlate with "good outcomes" on average; it must track the marginal value of acquiring information in each posterior state induced by the dialogue.

Limitations and scope. Our model is deliberately stylized. Real assistants may issue multiple tool calls, interleave clarifying questions with retrieval, face constraints from safety policies, and serve heterogeneous users with different cost sensitivities. We view the present framework as a baseline that isolates a core failure mode: mis-optimized information acquisition under modular training. The unbounded inefficiency result is an existence theorem, not a claim that all deployed systems are arbitrarily bad. Its role is diagnostic: it warns that without explicit alignment between the stage-1 proxy

and user value-of-information, there is no general welfare guarantee—even if the answer generator is excellent. Conversely, the bounded-loss results provide a way to recover guarantees under auditable conditions, pointing toward training and evaluation protocols that focus on value-of-information estimation.

Roadmap. We next present the formal model, defining the latent intent, the dialogue-derived signal, the tool-use action (including query choice), the induced information signal, and the final answer. We then compare an end-to-end user-optimal policy to modular pipelines, establish the unbounded inefficiency construction, and finally state conditions under which a proxy that certifies value-of-information yields bounded welfare loss.

2. Model. We model a single user–assistant interaction as a Bayesian decision problem with endogenous information acquisition. The assistant observes a dialogue-derived signal and then chooses whether to call an external tool ("search") and, if so, how to parameterize that call (a query template). The tool call produces an additional signal (e.g., retrieved snippets), after which the assistant commits to a final answer. Our goal in this section is to make explicit (i) what is uncertain, (ii) what the assistant observes and controls, and (iii) how user welfare depends on these objects.

Latent intent and dialogue signal. There is a latent user intent/state $\theta \in \Theta$ drawn from a prior distribution D over a measurable space $(\Theta, \mathcal{F})$. We interpret θ broadly: it captures the factual state relevant to the user's question as well as contextual elements that affect what constitutes a good answer (e.g., jurisdiction, time sensitivity, preferred level of detail, constraints such as "do not use X"). The assistant does not observe θ directly. Instead it observes a dialogue-derived signal $x \in \mathcal{X}$ (the conversation so far, possibly including system-side features such as locale or device type). Formally, we assume x is generated according to some conditional distribution $P(\cdot \mid \theta)$ on $(\mathcal{X}, \mathcal{A})$, inducing a posterior over latent intents,

$$\mu(\cdot \mid x) \;\equiv\; \Pr(\theta \in \cdot \mid x).$$

We will typically reason conditional on x; thus x is the sufficient statistic summarizing everything the assistant knows before choosing whether to search.

Tool-use action and query choice. After observing x, the assistant chooses a *tool-use action*. In the baseline model there are two actions: do not call an external tool, or call search. To allow the assistant to influence what is learned from search, we additionally allow it to pick a query parameterization. Concretely, let

$$s \in \mathcal{S} \;\equiv\; \{\mathsf{NoSearch}, \mathsf{Search}\}, \qquad q \in \mathcal{Q} \cup \{\varnothing\}.$$

If $s = \mathsf{NoSearch}$, we interpret $q = \varnothing$ as a null query. If $s = \mathsf{Search}$, then $q \in \mathcal{Q}$ is selected by the assistant and can represent a literal query string, a query template with slots filled from x, a retrieval configuration (e.g., corpus choice, freshness filter), or a structured tool call. Allowing q is important because "search" is not a single information structure: different query formulations induce different conditional distributions over what evidence is returned.

We sometimes write the stage-1 decision as $a \equiv (s, q)$, with feasible set

$$\mathcal{A}_1 \equiv \{(\mathsf{NoSearch}, \varnothing)\} \cup \{(\mathsf{Search}, q) : q \in \mathcal{Q}\}.$$

Signal generation as an information structure. Given latent intent θ and the assistant's stage-1 choice (s, q), an information signal σ is generated. Let $(\Sigma, \mathcal{G})$ denote the measurable signal space. For each feasible pair (s, q), we specify a Markov kernel

$$Q_{s,q}(\cdot \mid \theta) \in \Delta(\Sigma),$$

interpreted as the distribution of the observed tool output conditional on θ. If $s = \mathsf{Search}$, then σ might encode retrieved documents, snippets, rankings, timestamps, or other metadata. If $s = \mathsf{NoSearch}$, we can model the absence of a tool call as a degenerate signal $\sigma = \perp$ by letting Σ include a distinguished null element $\perp$ and setting $Q_{\mathsf{NoSearch},\varnothing}(\perp \mid \theta) = 1$ for all θ.

This formulation is deliberately agnostic about the mechanics of retrieval: all such details enter only through the induced conditional distribution $Q_{s,q}(\cdot \mid \theta)$. It also highlights the experiment-design view: by choosing (s, q), the assistant chooses which conditional distribution of σ will be observed.

Final answer decision. After σ is realized, the assistant chooses a final output y from an action space $\mathcal{Y}$ (answers, including possibly a refusal, a set of citations, or a structured response). The assistant's decision rule at this stage can depend on both x and σ, since x may carry context that remains relevant even after search. Formally, the stage-2 mapping is a measurable function

$$\pi_2 : \mathcal{X} \times \Sigma \to \mathcal{Y}, \qquad y = \pi_2(x, \sigma),$$

or, allowing randomized policies, a Markov kernel from (x, σ) to $\Delta(\mathcal{Y})$. Because our central comparative statics concern the *information acquisition* step, we allow the answer space $\mathcal{Y}$ to be rich (e.g., free-form text) but do not model language generation explicitly; instead we treat y as the decision variable that affects utility.

User utility and search costs. User welfare depends on how well y matches the latent intent and on the cost of tool use. Let

$$u : \Theta \times \mathcal{Y} \to \mathbb{R}$$

denote user utility from receiving answer y when the true state is θ. This utility can encode factual correctness, completeness, helpfulness, and adherence to constraints; it can also incorporate penalties for overconfidence or for presenting stale information. We assume u is measurable, and in several results we will additionally impose boundedness, e.g. $u \in [0, 1]$, to obtain uniform welfare bounds.

Tool use induces an additive cost. In the simplest case, the cost depends only on whether search is called:

$$c : \mathcal{S} \to \mathbb{R}_{\geq 0}, \qquad c(\mathsf{NoSearch}) = 0, \quad c(\mathsf{Search}) = c > 0.$$

More generally, one can allow query-dependent costs $c(s, q)$ (e.g., calling a premium corpus, requesting high-freshness results, or issuing a longer query) or even cost that depends on realized outcomes (e.g., requiring a follow-up click). For most of the paper we keep costs ex ante and action-dependent to isolate the welfare implications of choosing the information structure.

Given x, action (s, q), realized signal σ, and final answer y, we define *interim* user welfare as

$$W(x, s, q, \sigma, y) \;\equiv\; \mathbb{E}[u(\theta, y) \mid x, s, q, \sigma] \;-\; c(s),$$

where the conditional expectation is taken with respect to the posterior over θ induced by x and the likelihood $Q_{s,q}(\sigma \mid \theta)$. This expression emphasizes the decision-theoretic nature of the problem: after observing (x, σ), the assistant chooses y to maximize posterior expected utility, but the decision to search affects the posterior itself and incurs cost.

Information structure and observability. A key distinction in our setting is what is observed by whom. The assistant observes x before choosing (s, q), and then observes σ if and only if it chooses $s = \mathsf{Search}$ (or $\sigma = \perp$ under $\mathsf{NoSearch}$ under our normalization). The user may observe some aspects of the tool-use decision indirectly (e.g., latency, or an explicit "Searching..." indicator), but in the baseline welfare definition the user's cost is captured by $c(s)$ and the user's benefit is captured by $u(\theta, y)$.

The *platform* (or training pipeline) may observe additional telemetry: whether a tool call was made, dwell time, clicks, or downstream engagement. These observations matter for how proxy objectives are constructed, but they do not enter user welfare directly. To allow discussion of misaligned training incentives, we optionally define a platform benefit function

$$B : \mathcal{X} \times \mathcal{S} \times (\mathcal{Q} \cup \{\varnothing\}) \times \Sigma \times \mathcal{Y} \to \mathbb{R},$$

which may depend on tool-call events, monetizable actions, or engagement. Importantly, B need not coincide with W and may be easier to measure. Later, stage-1 proxy objectives will be interpreted as functions derived from (or correlated with) B rather than W.

Policies and measurability. A (possibly randomized) end-to-end policy consists of two components: a stage-1 decision rule selecting (s, q) based on x, and a stage-2 decision rule selecting y based on (x, σ). Formally, we can write an end-to-end policy as

$$\pi \equiv (\pi_1, \pi_2),$$

where π_1 is a measurable map from $\mathcal{X}$ to $\Delta(\mathcal{A}_1)$ and π_2 is a measurable map from $\mathcal{X} \times \Sigma$ to $\Delta(\mathcal{Y})$. Given π, the joint distribution over $(\theta, x, s, q, \sigma, y)$ is well-defined by the generative process: draw $\theta \sim D$, then $x \sim P(\cdot \mid \theta)$, then $(s, q) \sim \pi_1(\cdot \mid x)$, then $\sigma \sim Q_{s,q}(\cdot \mid \theta)$, then $y \sim \pi_2(\cdot \mid x, \sigma)$. We will evaluate policies by their expected user welfare,

$$\mathbb{E}_\pi[W] \;\equiv\; \mathbb{E}\Big[\; u(\theta, y) \;-\; c(s) \;\Big],$$

where the expectation is over all randomness in the environment and in the policy. (This equality follows from iterated expectations applied to our definition of $W(x, s, q, \sigma, y)$.)

We emphasize measurability because many of the comparisons we make are existence and worst-case statements. Our results do not rely on continuity or convexity in $\mathcal{Y}$; the key structure is the sequential revelation of information through $Q_{s,q}$.

Interpretation: where "retrieval quality" enters. In this model, the effectiveness of search is entirely governed by the family of kernels $\{Q_{\mathsf{Search},q}\}_{q \in \mathcal{Q}}$. If retrieval is noisy, biased, or systematically missing key sources, this is represented by σ being only weakly informative about θ (or informative in the wrong direction). Conversely, a high-quality retrieval system corresponds to an information structure in which σ tightly concentrates on evidence that identifies relevant aspects of θ. The point is not that retrieval is perfect; rather, the assistant's *choice* of whether/how to retrieve is a choice over these informational tradeoffs.

Extensions: multiple tool calls. Real assistants may issue multiple searches or combine tools (search, code execution, database queries). A natural extension is a finite-horizon sequential model. For $t = 1, \ldots, T$, the assistant chooses an action (s_t, q_t) based on the history $h_t = (x, \sigma_1, \ldots, \sigma_{t-1})$, observes $\sigma_t \sim Q_{s_t,q_t}(\cdot \mid \theta, h_t)$, and finally outputs y at time T (or chooses to stop endogenously). Costs become $\sum_{t=1}^{T} c(s_t)$, and welfare becomes expected utility of the final answer minus total cost. Our baseline one-shot model can be viewed as the case $T = 1$, which already captures the central externality of modularity: an upstream choice determines what information is available downstream. Allowing $T > 1$ introduces dynamic programming structure and makes the space of information acquisition policies richer, but it does not remove the core possibility that a mis-optimized early-stage proxy locks the system into a low-welfare information trajectory.

Extensions: clarifying questions as information acquisition. Clarifying questions can be treated as another information acquisition action alongside search. One way to incorporate them is to expand the stage-1 action set to include $s = \mathsf{Ask}$ with a question design variable $q \in \mathcal{Q}_{\mathsf{ask}}$, where q specifies what to ask. The resulting signal σ is then the user's reply, distributed according to a kernel $Q_{\mathsf{Ask},q}(\cdot \mid \theta)$. Asking has its own cost (interaction friction, abandonment risk) and can be complementary with search (e.g., ask to resolve ambiguity, then search with a more precise query). Framing clarifications as information structures emphasizes that "ask" and "search" are comparable in principle: both are experiments that trade off informativeness against cost, and both can be mis-triggered by proxy-trained modules.

Discussion and scope. This model abstracts away from several important operational details: it does not explicitly represent safety constraints, adversarial content, or the internal reasoning process by which the assistant converts σ into y. These elements matter in practice, but for our purposes they can be incorporated into $u(\theta, y)$ (e.g., by assigning low utility to unsafe outputs) and into $Q_{s,q}$ (e.g., by allowing σ to include misleading sources). What we retain is the minimal structure needed to study the central question: how does separating information acquisition (search triggering and query choice) from downstream decision-making interact with proxy objectives, and when can such modularity be justified in terms of user welfare?

3. Benchmarks: End-to-End Optimality and Value of Information. Our welfare benchmark is the policy that optimizes the user's objective *end-to-end*, taking into account both (i) how information acquisition changes what can be inferred about the latent intent and (ii) the cost of acquiring that information. This benchmark is standard in Bayesian decision theory, but spelling it out is useful because it makes explicit the quantity that any stage-1 tool-use module is implicitly trying (and often failing) to approximate: the *incremental value of the signal* generated by search.

End-to-end Bayes-optimal policy. Fix the primitives from Section 2. An end-to-end policy $\pi = (\pi_1, \pi_2)$ induces a joint distribution over $(\theta, x, s, q, \sigma, y)$ and hence an expected welfare $\mathbb{E}_\pi[u(\theta, y) - c(s)]$. We define the user-optimal policy as the Bayes-optimal solution to this sequential decision problem:

$$\pi^\star \in \arg\max_{\pi} \ \mathbb{E}_\pi[u(\theta, y) - c(s)] \,.$$

While the policy class allows randomization, the problem has the familiar dynamic-programming structure: after (x, σ) is observed, the assistant should choose the action y that maximizes posterior expected utility; anticipating this, at the moment of choosing (s, q) the assistant should trade off

the expected improvement in that posterior-optimal utility against the cost of searching.

To make this precise, define the *posterior-optimal (stage-2) value* after observing (x, σ) as

$$V_2(x, \sigma) \equiv \sup_{y \in \mathcal{Y}} \mathbb{E}[u(\theta, y) \mid x, \sigma]. \tag{1}$$

Under mild regularity (e.g., $\mathcal{Y}$ compact and $u(\theta, y)$ continuous in y for each θ, or $\mathcal{Y}$ countable), the supremum is attained, and the optimal stage-2 decision rule can be taken as a measurable selector

$$y^\star(x, \sigma) \in \arg\max_{y \in \mathcal{Y}} \mathbb{E}[u(\theta, y) \mid x, \sigma].$$

This object captures an idealized assumption we will frequently use later: *conditional on whatever evidence is available*, the assistant answers as well as possible for the user. Any inefficiency we highlight will come from selecting the wrong evidence to acquire, not from misusing the evidence once obtained.

Ex ante continuation value of a tool-use action. Consider a fixed dialogue signal realization x. If the assistant chooses a stage-1 action $a = (s, q)$, it induces a conditional distribution over the subsequent signal σ via the kernel $Q_{s,q}(\cdot \mid \theta)$ and the posterior over θ given x. The relevant object for stage-1 choice is therefore the *ex ante* expected stage-2 value, net of cost:

$$V_1(x; s, q) \equiv \mathbb{E}[\, V_2(x, \sigma) \mid x, s, q \,] - c(s). \tag{2}$$

The conditional expectation integrates over $\theta \sim \mu(\cdot \mid x)$ and then $\sigma \sim Q_{s,q}(\cdot \mid \theta)$. The Bayes-optimal stage-1 choice at x is any maximizer of $V_1(x; s, q)$:

$$(s^\star(x), q^\star(x)) \in \arg\max_{(s,q) \in \mathcal{A}_1} V_1(x; s, q),$$

with the convention that $q^\star(x) = \varnothing$ whenever $s^\star(x) = \mathsf{NoSearch}$. Combining these two stages yields an end-to-end optimal policy of the form

$$\pi^\star(x, \sigma) = \big(s^\star(x), q^\star(x), y^\star(x, \sigma)\big),$$

where $y^\star$ is computed under the posterior induced by the chosen action.[1]

[1] When we allow randomized policies and measurability subtleties, $\pi^\star$ should be interpreted as a measurable maximizer of $\mathbb{E}_\pi[u - c]$; the dynamic program above provides the standard characterization. The subsequent results in the paper concern comparisons to this benchmark and do not hinge on uniqueness.

The canonical "search iff value exceeds cost" rule. The expression (2) yields the familiar threshold structure for information acquisition. To see it cleanly, separate the no-search baseline from the search value.

If the assistant does not search, the signal is degenerate ($\sigma = \perp$), so the continuation value is simply

$$V^{\mathsf{No}}(x) \;\equiv\; V_1(x; \mathsf{NoSearch}, \varnothing) \;=\; \sup_{y \in \mathcal{Y}} \mathbb{E}[u(\theta, y) \mid x]\,. \tag{3}$$

If the assistant searches using query q, then it obtains a (generally informative) signal σ and can tailor y to that realization:

$$V^{\mathsf{Search}}(x; q) \;\equiv\; V_1(x; \mathsf{Search}, q) \;=\; \mathbb{E}\left[\sup_{y \in \mathcal{Y}} \mathbb{E}[u(\theta, y) \mid x, \sigma, \mathsf{Search}, q] \,\middle|\, x, \mathsf{Search}, q \right] - c. \tag{4}$$

(When costs are query-dependent, replace c by $c(\mathsf{Search}, q)$; nothing essential changes.)

Define the *incremental value of information* (VoI) of searching with query q at dialogue state x as the improvement in expected posterior-optimal utility before paying the cost:

$$\Delta\mathrm{VoI}(x; q) \;\equiv\; \mathbb{E}\left[\sup_{y \in \mathcal{Y}} \mathbb{E}[u(\theta, y) \mid x, \sigma, \mathsf{Search}, q] \,\middle|\, x, \mathsf{Search}, q \right] - \sup_{y \in \mathcal{Y}} \mathbb{E}[u(\theta, y) \mid x]\,. \tag{5}$$

This quantity is always weakly nonnegative under our formulation: with an extra signal in hand, the assistant can always ignore it and reproduce the no-search action, so the optimal expected utility cannot decrease (this is the standard monotonicity of information).[2]

With (5), we can rewrite the search value (4) succinctly:

$$V^{\mathsf{Search}}(x; q) \;=\; V^{\mathsf{No}}(x) \;+\; \Delta\mathrm{VoI}(x; q) \;-\; c.$$

Hence the Bayes-optimal decision rule takes the threshold form

$$\text{choose } \mathsf{Search} \text{ with some } q \quad\Longleftrightarrow\quad \max_{q \in \mathcal{Q}} \Delta\mathrm{VoI}(x; q) \;\geq\; c, \tag{6}$$

and otherwise choose $\mathsf{NoSearch}$. When the query set is singleton (or when the system uses a fixed query template), we can drop q and define $\Delta\mathrm{VoI}(x) \equiv \Delta\mathrm{VoI}(x; q)$, yielding the most familiar statement: *search iff the incremental value of the expected evidence exceeds the search cost.*

[2]Formally, $V_2(x, \sigma) \geq V^{\mathsf{No}}(x)$ pointwise if $\mathcal{Y}$ and the posterior are unaffected by conditioning on σ, because the no-search maximizer remains feasible after observing σ. This is the decision-theoretic "value of information is nonnegative" result, not a claim that any particular retrieval system improves utility in practice; failures of retrieval quality enter via how informative or misleading $Q_{s,q}$ is about θ, which affects the magnitude of (5).

Interpretation: what $\Delta\mathrm{VoI}(x)$ measures in assistant terms. Equation (5) makes clear what the tool-use decision is optimizing under the benchmark: not a generic preference for "using tools," but the expected gain from being able to *condition the answer* on the additional evidence produced by that tool call. This gain is high when (i) the posterior under x leaves materially relevant uncertainty about θ and (ii) the induced signal σ is informative about precisely those uncertain aspects of θ that matter for choosing a high-utility y. Conversely, $\Delta\mathrm{VoI}(x; q)$ is small when the question is already answered well from the dialogue alone, when search is likely to return redundant evidence, or when the best response y is insensitive to the remaining uncertainty.

This framing also clarifies the role of query choice q: different query templates implement different *experiments* (different information structures), and (6) says that the assistant should select the experiment with the highest incremental decision value. In other words, under the welfare benchmark, query formulation is not primarily about matching user keywords; it is about maximizing expected downstream utility subject to the cost of invoking search.

Regularity and measurability remarks. The threshold characterization above is purely decision-theoretic and requires little structure beyond well-defined conditional expectations. Two technical points are worth noting. First, the definition of $V_2(x, \sigma)$ as a supremum is compatible with rich answer spaces; we only need that $u(\theta, y)$ is measurable and integrable so that $\mathbb{E}[u(\theta, y) \mid \cdot]$ exists. When we later want *uniform* approximation guarantees (bounds that hold across all x), we will impose boundedness, e.g. $u \in [0, 1]$, which implies $\Delta\mathrm{VoI}(x; q) \in [0, 1]$. Second, the maximization over queries in (6) is well-behaved if $\mathcal{Q}$ is finite (as in many engineered systems where the query template is chosen from a menu), or more generally if standard measurable selection conditions hold. In such cases the stage-1 optimal policy can be taken to be (essentially) deterministic given x, with randomization only on measure-zero tie sets.

A useful decomposition: "quality of evidence" versus "value for this user state." A recurring source of confusion in practice is to equate "search is good" with a static notion of retrieval quality. The benchmark separates two distinct issues:

1. *Intrinsic informativeness of the channel* $Q_{\mathsf{Search},q}$: how much does the returned σ vary with θ and how reliably does it reflect the relevant aspects of θ?

2. *Decision sensitivity at* x: even if σ is informative, does that information change what answer is optimal? This is captured by the gap between optimizing after observing σ and optimizing before observing it.

The value-of-information term $\Delta\text{VoI}(x; q)$ bundles both elements: it is large only when the information is both available *and* decision-relevant at the current posterior induced by x. This is why a welfare-optimal assistant may rationally decline to search even when search is generally accurate: if the posterior already concentrates on a single high-utility response, the marginal benefit of additional evidence is low.

Connection to Bayesian experiment design. It is often helpful to restate (5) as an experiment-selection problem. Fix x and view the family $\{Q_{\mathsf{Search},q}\}_{q\in\mathcal{Q}}$ as a menu of experiments that map θ into observable signals. The benchmark chooses the experiment maximizing expected utility improvement net of cost. This is the standard Bayesian value-of-information criterion, but with two practical twists relevant for assistants: (i) the "experiment" includes query parameterization and retrieval configuration, not just a binary search/no-search flag, and (ii) the downstream decision is an answer in a rich language action space, which we treat abstractly as $\mathcal{Y}$.

This perspective will matter when we compare to modular training: any stage-1 module that is trained on a proxy not tightly linked to (5) is, in effect, selecting experiments using the wrong objective. Our welfare benchmark is therefore not merely an idealized target; it is also the conceptual object against which we will measure the *direction* and potential *magnitude* of distortions introduced by proxy-based triggering.

Summary of the benchmark. For each dialogue state x, the end-to-end benchmark reduces the tool-use decision to a single statistic: the incremental value of being able to condition the answer on the additional signal produced by search. The optimal policy searches (and selects a query) exactly when this incremental value exceeds the user's cost of search, as in (6). In the next section, we will use this benchmark as the reference point for formalizing modular training pipelines and for diagnosing how proxy-optimized stage-1 decisions can depart from the value-of-information criterion even when the downstream answer module behaves optimally conditional on the information it receives.

4. A Class of Modular Training Pipelines. We now formalize a broad family of engineering workflows in which the assistant's *information acquisition* decision is trained separately from its *answering* behavior. The key modeling move is to treat the tool-use decision as a stage-1 module that optimizes a proxy objective, while the response module is a stage-2 optimizer that (possibly very well) uses whatever evidence it is handed. This captures a common division of labor in deployed systems: a lightweight trigger (and sometimes query generator) decides whether to call an external tool, and a separate retrieval-augmented generator (RAG) or answer model produces the final output conditioned on the retrieved content.

Our goal in this section is not to argue that modular pipelines are "irrational"—they are often the only practical choice given latency budgets, observability constraints, and the difficulty of defining a single end-to-end reward. Rather, we want a clean abstraction that (i) maps recognizable training choices into a decision-theoretic object and (ii) makes it transparent how optimizing the wrong stage-1 proxy can systematically select the wrong information structure, even if the downstream answerer is essentially optimal given the evidence it receives. This sets up the welfare comparisons and worst-case constructions in the next section.

Stage-1 and stage-2 as separate maximizations. Fix the primitives from our model: the dialogue-derived signal x, a stage-1 action consisting of a tool-use flag and query template $a = (s, q) \in \mathcal{A}_1$ with $s \in \{\mathsf{NoSearch}, \mathsf{Search}\}$ and $q \in \mathcal{Q}$ (with $q = \varnothing$ when $s = \mathsf{NoSearch}$), a subsequent tool signal σ drawn from the induced kernel, and a final answer $y \in \mathcal{Y}$. In an end-to-end benchmark, stage-1 would trade off the cost of search against the incremental value of conditioning y on σ. A modular pipeline, by contrast, makes stage-1 a best response to an auxiliary training signal.

We represent this by two objectives:

$$J_1(s, q \mid x) \quad \text{(stage-1 proxy for choosing whether/how to search)}, \tag{7}$$

$$J_2(y \mid x, s, q, \sigma) \quad \text{(stage-2 objective for producing the final answer)}. \tag{8}$$

The modular pipeline chooses (s, q) as

$$g(x) \in \arg \max_{(s,q) \in \mathcal{A}_1} J_1(s, q \mid x), \tag{9}$$

then chooses the answer as

$$h(x, s, q, \sigma) \in \arg \max_{y \in \mathcal{Y}} J_2(y \mid x, s, q, \sigma). \tag{10}$$

The induced policy is therefore

$$\pi^{\mathrm{mod}}(x, \sigma) = \big(g(x),\, h(x, g(x), \sigma)\big), \tag{11}$$

where we suppress the explicit dependence of h on (s, q) when clear. We allow g and h to be randomized or set-valued (ties), but the welfare pathologies we study do not rely on such subtleties.

Two remarks clarify what this abstraction includes. First, J_2 can coincide with user welfare *conditional on evidence* (e.g., it can be calibrated to maximize $\mathbb{E}[u(\theta, y) \mid x, s, q, \sigma]$), and we will often impose this as a "best case" for modularity: any inefficiency then comes purely from stage-1 choosing the wrong information structure. Second, we do *not* assume that the proxy J_1 is adversarial; it may be a reasonable operational metric (latency, clicks,

satisfaction ratings) or a learned reward model. The point is that these metrics are rarely equal to the Bayesian value-of-information expression from Section 3, and the wedge between them is precisely what modular training introduces.

Mapping common system designs into (J_1, J_2). The stage-1 module g can be understood as any mechanism that gates tool access. In practice it is often one of the following.

(i) A tool-use classifier trained on labels. A common approach is to train a classifier on logged conversations with a binary label "search was used" or "search was helpful." This corresponds to choosing a proxy

$$J_1(\mathsf{Search} \mid x) = \Pr(\text{label} = 1 \mid x), \qquad J_1(\mathsf{NoSearch} \mid x) = 0,$$

or a cost-adjusted variant. When labels are derived from human heuristics ("current events $\Rightarrow$ search") or from retrospective judgments ("search improved factuality"), the classifier inherits the bias and noise of that labeling rule. Crucially, even an unbiased label of "search was used" is not a label of "search's incremental value net of cost."

(ii) A discrete policy optimized by reinforcement learning on platform metrics. Many deployments optimize stage-1 using RL (or contextual bandits) against measurable outcomes such as click-through, dwell time, or ad revenue. This falls directly into (9), with J_1 the learned Q-value for a proxy reward. Importantly, the state for this RL problem is often x (possibly plus some shallow features), not the latent intent θ; moreover, the reward is typically a platform metric, not the user utility $u(\theta, y)$.

(iii) Query selection as a separate learned component. Even if the system always searches, many architectures still choose among query templates, retrieval indices, or tool configurations. Our $q \in \mathcal{Q}$ is meant to subsume these "experiment design" knobs. For example, $\mathcal{Q}$ might index (a) a "freshness-first" query for news, (b) a "high-precision" query that narrows to official sources, or (c) a "broad" query that maximizes recall. Training q against a proxy (e.g., maximizing clicks on retrieved results) is again an instance of (9).

On the stage-2 side, h corresponds to what is usually called the answer model (often an LLM) together with the prompt and decoding rules. Typical J_2 include supervised likelihood of reference answers, preference-model rewards for helpfulness, and factuality/scoring objectives that are evaluated *conditional* on the retrieved documents. When the stage-2 module is a strong conditional optimizer, it is natural to approximate it as "pointwise optimal given (x, s, q, σ)," which is exactly the assumption we will use to isolate the welfare effect of stage-1 mis-triggering.

Why modularity is an assumption about *optimization*, not architecture. It is tempting to equate "modular" with "two neural networks."

We mean something slightly different: modularity is the *separation of objectives* across the information acquisition step and the answering step. A system can be architecturally end-to-end (one model) but still modular in our sense if it is trained with auxiliary losses that push tool-use behavior toward a proxy reward while training answering behavior toward correctness conditional on retrieval. Conversely, a system can be architecturally modular but trained end-to-end with a single welfare-aligned objective (in which case it would not fit our modular class). We focus on the former because it is operationally common: stage-1 behavior is often tuned using abundant telemetry, while stage-2 is tuned using smaller, higher-quality supervision.

Concrete instantiation 1: monetization-weighted proxy. We first instantiate J_1 with a stylized but realistic pattern: stage-1 is trained to maximize a combination of user-facing metrics and monetizable events. Let M be an observable monetization outcome (ad impression, sponsored click, referral, etc.) realized after the stage-1 action and downstream interaction. In many platforms, tool calls create additional "inventory" for such events (e.g., a search results page with sponsored slots), or they change the probability of downstream transactions. A reduced-form stage-1 proxy can therefore be written as

$$J_1(\mathsf{Search}, q \mid x) = \lambda \cdot \mathbb{E}[M \mid x, \mathsf{Search}, q] \; + \; \beta \cdot \mathbb{E}[S \mid x, \mathsf{Search}, q] \; - \; \kappa, \quad (12)$$

with

$$J_1(\mathsf{NoSearch}, \varnothing \mid x) = 0.$$

Here S can represent a coarse "user satisfaction" signal available at scale (thumbs-up rate, complaint probability with negative sign, etc.), λ is the monetization weight, β the satisfaction weight, and κ a fixed penalty capturing latency or operational cost (often set too low when the platform internalizes only part of the user cost). The particular form is not essential; what matters is that the proxy is a weighted sum of measurable outcomes rather than the user's incremental value of information.

This proxy can generate both over-search and under-search relative to the welfare benchmark.

- *Over-search:* if λ is large and tool invocation mechanically increases M (e.g., by generating an impression), then (12) can favor Search even when the expected answer-quality gain is negligible or when the tool results are noisy. From the user's perspective, this is precisely the regime where $\Delta\mathrm{VoI}(x; q)$ is small but the platform proxy is large.

- *Under-search:* if informative searches reduce monetization (e.g., because a correct answer resolves the user's need quickly and reduces additional browsing), then $\mathbb{E}[M \mid x, \mathsf{Search}, q]$ may be *lower* for the

welfare-improving queries. In that case, stage-1 may systematically avoid precisely the information structures that most improve accuracy, because those structures reduce proxy value.

This aligns with the incomplete contracting logic: the platform can write contracts (and therefore optimize) on M and on coarse S, but cannot directly contract on $u(\theta, y)$ for every latent user intent. Optimizing (12) is then a rational organizational choice that nonetheless induces misalignment.

Concrete instantiation 2: engagement proxy. A second common proxy family optimizes for interaction volume or session-level engagement. Let E denote an engagement measure such as number of turns, dwell time, or probability the user asks a follow-up. A simple proxy is

$$J_1(\mathsf{Search}, q \mid x) = \alpha \cdot \mathbb{E}[E \mid x, \mathsf{Search}, q] \; - \; \kappa, \qquad J_1(\mathsf{NoSearch}, \varnothing \mid x) = 0. \tag{13}$$

This proxy is attractive because E is easy to measure and often correlates with some notions of satisfaction in aggregate. But its relationship to user welfare is ambiguous at the margin. Informative searches can reduce engagement by enabling a one-shot correct response (a user who gets the right citation immediately may leave), whereas uninformative searches can increase engagement by producing confusion, prompting clarification questions, or encouraging repeated reformulations. Thus, much like the monetization case, engagement can be negatively correlated with the value of information.

Moreover, engagement proxies can distort *query choice* even when the binary search decision is correct. If some queries produce long, ambiguous, or contradictory evidence (raising the chance of follow-up interaction), then maximizing (13) can steer the system toward those queries, away from high-precision experiments that would collapse uncertainty quickly. In our notation, this is a distortion in the argmax over $q \in \mathcal{Q}$ inside $g(x)$.

Stage-2 objectives: correctness conditional on the chosen evidence. To isolate the effect of stage-1, we will often place stage-2 in an optimistic regime. Concretely, we can take

$$J_2(y \mid x, s, q, \sigma) = \mathbb{E}[u(\theta, y) \mid x, s, q, \sigma]\,, \tag{14}$$

so that h is exactly the posterior-optimal response given the realized signal. This captures a design intent behind many RAG systems: "given retrieved documents, answer as accurately/helpfully as possible." It also matches training pipelines where the answer model is evaluated on factuality and citation quality conditional on retrieval outputs.

We stress, however, that treating stage-2 as optimal is a modeling convenience, not a claim about current systems. Retrieval can be misleading, models can hallucinate, and user utility is richer than factual correctness.

Importantly, these imperfections only strengthen the motivation for analyzing the stage-1 decision carefully: if stage-2 were imperfect, bad information acquisition could be even more harmful. Our negative results will therefore be robust in the sense that they do not rely on stage-2 mistakes.

Data separation and the source of misalignment. One practical reason modularity persists is that the two objectives are trained on different data streams. Stage-1 has abundant implicit feedback (search events, clicks, latency, session continuation), while stage-2 relies on comparatively scarce high-quality supervision (expert labels, preference comparisons, red-teaming). Our abstraction captures this by allowing J_1 and J_2 to be learned from different distributions and to encode different stakeholders' priorities. The result is not merely estimation error; it is a structural wedge: even with infinite data, if J_1 differs from the user's incremental value of information, $g(x)$ can converge to a systematically distorted information acquisition policy.

This is exactly the sense in which modularity is nontrivial: the welfare benchmark cares about selecting an information structure (search/no-search and query) that maximizes decision value net of cost, whereas the modular stage-1 module may be trained to maximize a proxy that is only loosely related (and can be negatively related) to that decision value.

Summary and what we will prove next. We have defined a large class of modular pipelines via the two best-response conditions (9)–(10) and illustrated how common engineering choices instantiate J_1 as monetization- or engagement-weighted proxies. This class is broad enough to capture "reasonable" tool-use triggers (classifiers, bandits, RL policies) while still being sharp enough to analyze formally.

In the next section, we will use this abstraction to show that proxy-optimized stage-1 selection can yield *unbounded* inefficiency relative to the end-to-end welfare benchmark: even when stage-2 is (conditionally) user-optimal, choosing the wrong information structure at stage-1 can destroy welfare. We will present explicit constructions exhibiting both over-search and under-search failures, as well as a query-steering variant where the system searches but systematically chooses the wrong query template.

5. Main Negative Result: Unbounded Price of Anarchy. This section formalizes a stark message: even if the stage-2 answerer is as good as one could hope for (it always chooses the user-optimal response given the evidence it receives), a stage-1 proxy that is even slightly misaligned with the user's *incremental value of information* can drive arbitrarily large welfare losses. The economic intuition is straightforward. Stage-1 is not "just" a binary classifier; it is an *experiment selection* problem. If the proxy pushes the system toward the wrong experiment (no experiment at all, a

noisy one, or the wrong query template), then the downstream module is optimizing the wrong posterior, and no amount of conditional optimality at stage-2 can recover the lost value.

We state this as an unbounded Price-of-Anarchy (PoA) result: the ratio between the end-to-end optimal welfare and the modular welfare can be made arbitrarily large by an explicit family of instances. The constructions mirror the mechanism in Bhawalkar–Psomas–Wang (2025): selecting an information structure via an auxiliary objective can be arbitrarily inefficient relative to the welfare that depends on the *allocation* (here: the final answer) induced by that information.

Benchmark ratio. For an instance, define the modular inefficiency ratio

$$\mathrm{PoA} \;\equiv\; \frac{\mathbb{E}[W(\pi^{\star})]}{\mathbb{E}[W(\pi^{\mathrm{mod}})]},$$

whenever the denominator is positive. (We construct instances where it is positive and can be made arbitrarily small, so the ratio is well-defined and large.)

Theorem 0.1 (Unbounded modular Price of Anarchy)**.** *Fix any constant $C > 0$. There exists an instance (a distribution over intents D, tool signal kernels, utility u, costs c, query set $\mathcal{Q}$, and objectives (J_1, J_2)) such that:*

1. *the stage-2 module is pointwise user-optimal given evidence, i.e.,*

$$J_2(y \mid x, s, q, \sigma) = \mathbb{E}[u(\theta, y) \mid x, s, q, \sigma] \quad \Rightarrow \quad h \in \arg\max_{y} \mathbb{E}[u(\theta, y) \mid x, s, q, \sigma],$$

2. *the induced modular policy π^{mod} satisfies $\mathbb{E}[W(\pi^{\mathrm{mod}})] > 0$, and*
3. $\mathrm{PoA} \geq C$.

Moreover, the lower bound holds under each of three distinct failure modes: (i) under-search *(stage-1 declines valuable search), (ii)* over-search *(stage-1 triggers worthless search with near-unit cost), and (iii)* query-steering *(stage-1 searches but selects the wrong query template).*

Proof idea (before formal details). Each construction is parameterized by a scalar V that controls how valuable the *right* information structure is. The end-to-end policy uses that structure and achieves welfare on the order of V. The modular policy, however, is pushed by J_1 toward an action whose resulting signal is uninformative (or toward no search at all), leaving stage-2 with no useful evidence; the best stage-2 can do then is fall back on a "safe" answer that yields welfare on the order of 1. By scaling V, we make the ratio arbitrarily large.

Construction A: Under-search (proxy discourages informative search)

We build an instance where search is extremely valuable for the user but is assigned a lower proxy score than not searching.

Primitives. Let $\Theta = \{0, 1\}$, with prior $\Pr(\theta = 0) = \Pr(\theta = 1) = 1/2$. Let the dialogue signal x be constant (so it conveys no information). There are three candidate answers $\mathcal{Y} = \{y_0, y_1, y_{\text{safe}}\}$. Fix a parameter $V > 1$ and define utility

$$u(\theta, y_{\text{safe}}) = 1 \quad \text{for both } \theta \in \{0, 1\},$$
$$u(\theta, y_\theta) = V, \qquad u(\theta, y_{1-\theta}) = 0.$$

Thus y_{safe} is a guaranteed-but-low-quality response, while y_θ is highly valuable if and only if we know the intent.

The stage-1 action set has two elements: NoSearch and Search (ignore q here by taking $\mathcal{Q}$ a singleton). If NoSearch, no signal is produced (equivalently $\sigma = \perp$). If Search, the tool returns a perfectly revealing signal $\sigma = \theta$.

Let costs be $c(\text{NoSearch}) = 0$ and $c(\text{Search}) = 0$ (we could add a small cost without changing the argument).

Stage-2 optimality. Given NoSearch, the posterior equals the prior, so the expected utility of y_{safe} is 1, while the expected utility of y_0 (or y_1) is $V/2$. To ensure stage-2 prefers the safe response under no information, we can either assume $V < 2$ for this sub-argument, or (more cleanly) slightly modify utilities so that $u(\theta, y_\theta) = V$ but the prior probability of each θ is so small that $V \Pr(\theta)$ is below 1. To keep the exposition simple while preserving scaling, take $\Pr(\theta = 0) = \Pr(\theta = 1) = 1/2$ but redefine the risky answers so that, absent evidence, the optimal action is the safe one; for example, introduce a small penalty $-\eta$ for choosing y_0 or y_1 when wrong. Equivalently, we can stipulate that the answer space only allows the safe response without evidence (a reduced-form "policy constraint"). In all cases the core mechanism is the same: without a good signal, stage-2 cannot reliably extract the large payoff V and therefore chooses the low baseline.

Given Search and $\sigma = \theta$, the posterior is degenerate, and stage-2 optimally chooses y_θ, achieving utility V.

Hence, with the optimistic stage-2 objective $J_2 = \mathbb{E}[u \mid \cdot]$, we have:

$$\mathbb{E}[W(\pi^\star)] = V, \qquad \mathbb{E}[W(\pi^{\text{mod}}) \mid \text{NoSearch}] = 1.$$

Stage-1 proxy. Define the proxy as

$$J_1(\text{Search} \mid x) = -1, \qquad J_1(\text{NoSearch} \mid x) = 0.$$

Then $g(x) = \text{NoSearch}$ uniquely, so the modular policy never searches, even though searching would allow stage-2 to realize the high-value action.

Welfare comparison. The end-to-end optimal policy searches and gets welfare V, while the modular policy does not and gets welfare 1. Therefore

$$\text{PoA} = \frac{V}{1} = V.$$

Choosing $V \geq C$ yields PoA $\geq C$. This proves Theorem 0.1 for the under-search failure mode. □

Construction B: Over-search (proxy rewards search even when it is useless)

We now flip the pathology: stage-1 searches because the proxy directly rewards tool calls, but the search signal is worthless and the cost is nearly as large as the entire baseline value, driving welfare arbitrarily close to zero.

Primitives. Keep $\Theta = \{0, 1\}$ with x constant and $\mathcal{Y} = \{y_{\text{safe}}, y_0, y_1\}$. Define utilities so that, absent any informative evidence, the safe action is optimal and yields value 1:

$$u(\theta, y_{\text{safe}}) = 1, \qquad u(\theta, y_0) = u(\theta, y_1) = 1$$

(or, more simply, restrict $\mathcal{Y} = \{y_{\text{safe}}\}$ so stage-2 always yields 1 regardless of beliefs). The key point is that information provides *no incremental value.*

Let Search produce an uninformative signal: σ is independent of θ and x (e.g., $\sigma \in \{0, 1\}$ fair coin). Thus, even with search, the posterior does not improve in any decision-relevant way. Set costs

$$c(\mathsf{NoSearch}) = 0, \qquad c(\mathsf{Search}) = 1 - \frac{1}{V},$$

with V a large parameter.

Stage-2 optimality. Because σ is useless, stage-2's optimal expected utility is 1 under either action. Thus,

$$\mathbb{E}[W \mid \mathsf{NoSearch}] = 1, \qquad \mathbb{E}[W \mid \mathsf{Search}] = 1 - \left(1 - \frac{1}{V}\right) = \frac{1}{V}.$$

The end-to-end optimal policy therefore chooses NoSearch and attains welfare 1.

Stage-1 proxy. Let the proxy reward tool usage mechanically:

$$J_1(\mathsf{Search} \mid x) = 1, \qquad J_1(\mathsf{NoSearch} \mid x) = 0,$$

so $g(x) = \mathsf{Search}$. This can be read as a reduced form of a monetization or "inventory creation" effect: a tool call produces an event the platform values, regardless of whether it improves the user outcome.

Welfare comparison. Then

$$\text{PoA} = \frac{1}{1/V} = V,$$

which exceeds C for $V \geq C$. This proves unboundedness for over-search. □

Construction C: Query-steering (search occurs, but the proxy selects the wrong information structure)

Finally, we show that inefficiency can arise even when the system always uses the tool: the proxy can distort *which* query template is selected, pushing the system toward a low-value experiment.

Primitives. Let $\Theta = \{0, 1\}$ with constant x and the same answer set as in Construction A. Suppose the system must pick between two query templates $q \in \{q_{\text{good}}, q_{\text{bad}}\}$, each with the same direct cost $c(\mathsf{Search}) = 0$ (costs could be equalized by engineering constraints). Under q_{good}, the signal reveals θ perfectly: $\sigma = \theta$. Under q_{bad}, the signal is independent of θ (pure noise). Utilities are as in Construction A, so that knowing θ is worth V while the safe fallback yields 1.

Stage-2 optimality. Given $(q_{\text{good}}, \sigma = \theta)$, stage-2 picks y_θ and obtains V. Given q_{bad}, stage-2 cannot infer θ and falls back to the safe response with utility 1.

Stage-1 proxy. Define a proxy that prefers the bad query template, for reasons orthogonal to answer quality (e.g., it yields more clicks, longer snippets, or higher engagement):

$$J_1(\mathsf{Search}, q_{\text{bad}} \mid x) = 1, \qquad J_1(\mathsf{Search}, q_{\text{good}} \mid x) = 0.$$

Then $g(x)$ selects q_{bad} deterministically.

Welfare comparison. The end-to-end policy chooses q_{good} and gets V, while the modular policy chooses q_{bad} and gets 1, hence $\text{PoA} = V$ as before. □

Discussion and limitations. Theorem 0.1 is intentionally worst-case. The proxies we wrote down are extreme, and the scaling parameter V effectively creates situations where the value of the *right* information structure is enormous. That is precisely the point: nothing in the modular decomposition prevents such instances, because the stage-1 objective is not, in general, constrained to be a calibrated estimate of the user's value of information.

In practice, the analogue of V can be "rare but high-stakes" intents (medical, legal, safety, finance) where the marginal value of correct grounding is very large, while the platform proxy may weakly penalize tool usage (latency budgets) or even reward it for unrelated reasons (inventory, engagement).

Equally important, the constructions show that the failure is *not* a matter of imperfect language modeling. We granted stage-2 the strongest possible property: posterior-optimal behavior given the evidence. The welfare loss is entirely attributable to the stage-1 selection of the information structure, echoing the broader lesson from information design: if the wrong experiment is chosen, even a perfect downstream decision rule is optimizing the wrong posterior.

This motivates the question taken up next: what structural restrictions on J_1 (and what calibration conditions on J_2) rule out these pathologies and yield uniform welfare guarantees?

6. Positive Results: When Modularity is Safe. The negative constructions hinge on a simple fact: stage-1 is choosing an *information structure*, so even a small systematic distortion in its objective can select the wrong experiment and thereby destroy downstream value. The natural way to rule this out is to require that the stage-1 proxy is not an arbitrary engagement or revenue predictor, but instead a *certificate* for the user's incremental value of information from searching. Under such a restriction, modularity becomes much closer to the end-to-end Bayes benchmark, and we can make this precise with both additive and (under mild regularity) multiplicative welfare guarantees.

6.1 The value of information as the normative stage-1 target

Fix a dialogue signal x. For each stage-1 action $a \in \mathcal{A}$, where $\mathcal{A} \equiv \{\mathsf{NoSearch}\} \cup (\{\mathsf{Search}\} \times \mathcal{Q})$, define the *user value* of committing to a and then behaving optimally at stage-2:

$$V(a \mid x) \;\equiv\; \mathbb{E}_{\sigma \sim Q_a(\cdot|x)}\Big[\max_{y}\; \mathbb{E}\big[u(\theta, y) \mid x, a, \sigma\big]\Big] - c(a),$$

where for $a = \mathsf{NoSearch}$ we take $\sigma = \perp$ deterministically and Q_a is degenerate. This quantity already "bakes in" the best-possible stage-2 response conditional on the information structure induced by a.

The end-to-end Bayes-optimal stage-1 choice at signal x is therefore

$$a^\star(x) \in \arg\max_{a \in \mathcal{A}} V(a \mid x),$$

and the incremental *value of information of searching* relative to not searching is the gap

$$\Delta\mathrm{VoI}(x) \;\equiv\; \max_{q \in \mathcal{Q}} V((\mathsf{Search}, q) \mid x) - V(\mathsf{NoSearch} \mid x).$$

In this language, the normative decision rule is: search (with an optimal query template) iff $\Delta\text{VoI}(x) \geq 0$.

Two remarks matter for modularity. First, the object stage-1 should approximate is *not* "probability the answer is wrong," nor "uncertainty," but the *decision-relevant* improvement in achievable utility net of costs. Second, because $V(a \mid x)$ already internalizes stage-2 optimal behavior, we can study stage-1 errors without separately modeling the language generation problem, as long as stage-2 is calibrated to implement the $\max_y$ above.

6.2 Value-of-information certificates

We formalize the restriction on the proxy J_1 as a uniform approximation (or a conservative lower bound) to $V(\cdot \mid x)$ or $\Delta\text{VoI}(x)$.

Two-sided certificates. We say J_1 is an (ϵ, δ) *value certificate* if, for all x and all $a \in \mathcal{A}$,

$$\big|J_1(a \mid x) - V(a \mid x)\big| \leq \epsilon,$$

and the stage-1 optimizer is allowed δ suboptimality, i.e., it outputs some $\hat{a}(x)$ satisfying

$$J_1(\hat{a}(x) \mid x) \;\geq\; \max_{a \in \mathcal{A}} J_1(a \mid x) - \delta.$$

This definition separates two common sources of modular slack: statistical estimation error (ϵ) and imperfect optimization or heuristics in the stage-1 policy (δ).

One-sided (conservative) certificates. In many deployments the larger concern is *over-search* (latency, privacy, annoyance). For that case, a one-sided condition is often more plausible:

$$J_1((\mathsf{Search}, q) \mid x) \;\leq\; V((\mathsf{Search}, q) \mid x) \quad \forall x, q, \qquad J_1(\mathsf{NoSearch} \mid x) = V(\mathsf{NoSearch} \mid x),$$

possibly up to additive ϵ. This guarantees that if stage-1 triggers search based on J_1, it is not doing so because of an "illusory" proxy gain. The tradeoff is that conservative certificates can induce under-search when the proxy fails to recognize some genuine value.

For clarity, we present the main welfare bounds under the two-sided notion; variants under one-sided certificates follow by similar (and often simpler) arguments.

6.3 Additive welfare guarantees

Assume stage-2 is *decision-calibrated*: for every (x, a, σ) it chooses

$$h(x, a, \sigma) \in \arg\max_y \; \mathbb{E}[u(\theta, y) \mid x, a, \sigma].$$

Then the welfare achieved by committing to stage-1 action a at signal x is exactly $V(a \mid x)$ by definition. Hence the welfare loss from modular stage-1 selection is the gap between the best $V(\cdot \mid x)$ and the chosen one.

Theorem 0.2 (Additive bound from value certificates)**.** *Suppose J_1 is an (ϵ, δ) value certificate and stage-2 is decision-calibrated. Let $a^\star(x) \in \arg\max_a V(a \mid x)$ be an end-to-end optimal stage-1 choice and let $\hat{a}(x)$ be the modular stage-1 choice. Then for every x,*

$$V(a^\star(x) \mid x) - V(\hat{a}(x) \mid x) \;\le\; 2\epsilon + \delta.$$

Consequently,

$$\mathbb{E}[W(\pi^\star)] - \mathbb{E}[W(\pi^{\mathrm{mod}})] \;\le\; 2\epsilon + \delta.$$

Why this is the right guarantee. The bound is uniform and *instance-independent*: it does not assume a margin separating "search" and "no-search" cases, nor any distributional structure beyond what is needed to define posteriors. Economically, it says that modularity is safe when the stage-1 proxy is a near-correct surrogate for the user's own net value function over experiments. The theorem also makes precise what the negative examples exploit: when J_1 can differ from V by an amount that scales with the stakes (our parameter V in the constructions), no uniform bound is possible.

Bounded utility range and randomized certificates. In applications, we rarely have a deterministic ϵ-uniform approximation. A common alternative is a high-probability or in-expectation guarantee. If utilities are bounded, say $u(\theta, y) \in [0, U]$ and costs are in $[0, U]$, then $V(a \mid x) \in [-U, U]$ and we can convert tail bounds on the certificate error into expected welfare bounds by standard concentration and union bounds over $\mathcal{A}$ (when $\mathcal{A}$ is finite), or via covering arguments (when $\mathcal{A}$ is infinite). The key role of boundedness is to prevent rare proxy failures from producing arbitrarily large welfare losses.

6.4 Multiplicative (ratio) guarantees and "margin" regularity

Additive guarantees are the natural benchmark in decision problems, but ratio guarantees are often easier to interpret as "approximate optimality." A ratio bound cannot hold without some regularity: if $\mathbb{E}[W(\pi^{\mathrm{mod}})]$ can be arbitrarily close to 0, then any fixed additive error yields an unbounded ratio.

One practical regularity is the existence of a baseline action whose welfare is uniformly bounded below. In conversational systems, the analogue is a "safe and helpful enough" response that achieves at least some $\underline{w} > 0$ for all x (e.g., a competent generic answer, a refusal with guidance, or a low-latency summary), and whose cost is controlled. Formally, suppose

$$V(\mathsf{NoSearch} \mid x) \ge \underline{w} \;> 0 \qquad \forall x.$$

Then the optimal welfare is also at least $\underline{w}$, and the additive bound translates into a ratio bound.

Corollary 0.3 (Ratio bound under a baseline floor)**.** *Under the assumptions of Theorem 0.2, if* $V(\mathsf{NoSearch} \mid x) \geq \underline{w} > 0$ *for all* x*, then*

$$\frac{\mathbb{E}[W(\pi^\star)]}{\mathbb{E}[W(\pi^{\mathrm{mod}})]} \;\leq\; 1 + \frac{2\epsilon + \delta}{\underline{w}}.$$

In particular, when $\underline{w}$ *is a constant and* ϵ, δ *are small, the ratio is* $1+O(\epsilon+\delta)$*.*

A different (and often more realistic) regularity is a *margin* condition: the set of x where $\Delta\mathrm{VoI}(x)$ is near zero has small probability mass. Then even a proxy with modest error will disagree with the end-to-end decision only on a small measure set, yielding a small expected welfare gap. This is the standard logic behind classification calibration, but here the "labels" are induced by an economic threshold, and the loss from misclassification is weighted by the magnitude of $\Delta\mathrm{VoI}(x)$.

6.5 Query templates: continuity and Lipschitz structure

When $\mathcal{Q}$ is large, the main new issue is uniformity over q. Theorem 0.2 continues to apply as stated if the certificate error is uniform over (x, q). When that is too strong, we can exploit structure.

A useful modeling assumption is that the family of information structures varies smoothly with q, so that $V((\mathsf{Search}, q) \mid x)$ is Lipschitz in an embedding of the query template space. Concretely, suppose $\mathcal{Q}$ is a compact metric space with distance $d(\cdot, \cdot)$ and for each x,

$$\left|V((\mathsf{Search}, q) \mid x) - V((\mathsf{Search}, q') \mid x)\right| \;\leq\; L\, d(q, q').$$

If we only learn J_1 accurately on an η-net of $\mathcal{Q}$, then the optimality loss from discretizing templates is at most $L\eta$, and the certificate error on the net adds in the same way as ϵ above. Operationally, this says: if neighboring query templates induce similar retrieval distributions and downstream utility, then coarse template classes (or a small learned set of tools/queries) are sufficient for near-optimal modular behavior.

This is also where "query steering" becomes architecturally addressable: if we constrain the allowable q to a low-complexity family where V is stable and auditable, the certificate problem becomes statistically and computationally easier.

6.6 Stage-2 calibration and proper scoring rules

All of the above treats stage-2 as implementing $\max_y \mathbb{E}[u(\theta, y) \mid \cdot]$. In practice, stage-2 is trained on proxy losses (next-token prediction, preference

modeling, etc.), so we need conditions under which those losses correspond to posterior expected utility maximization.

One clean route is to separate *belief* from *decision*. Suppose stage-2 produces (explicitly or implicitly) a predictive distribution $\hat{p}(\theta \mid x, a, \sigma)$, trained with a strictly proper scoring rule (e.g., log loss) on ground-truth θ (or on a sufficient statistic for utility). Proper scoring implies calibration: in the large-data limit, $\hat{p}$ matches the true posterior. Then a decision layer chooses

$$y \in \arg\max_{y} \ \mathbb{E}_{\theta \sim \hat{p}(\cdot \mid x,a,\sigma)}[u(\theta, y)],$$

which coincides with the Bayes action when $\hat{p}$ is correct. More generally, if u is bounded and Lipschitz in the posterior (under total variation or another divergence), then small miscalibration in $\hat{p}$ implies small regret in expected utility, yielding an additional additive term that composes with the $2\epsilon + \delta$ stage-1 bound.

This perspective clarifies what is (and is not) required for the positive results. We do *not* need stage-2 to be a perfect language model; we need it to be approximately Bayes-optimal for the *user utility-relevant* uncertainty, and we need the stage-1 proxy to be tied to the induced decision value.

6.7 Interpretation and limitations

The positive message is conditional but actionable: modularity is safe when the stage-1 objective is a faithful proxy for the user's net value of searching (including query choice), and when stage-2 is trained in a way that is decision-calibrated with respect to user utility. The bounds are deliberately simple; they are meant to function as "design inequalities" that tell us what must be controlled (proxy error and optimization slack) to prevent the unbounded failures.

At the same time, the assumptions are demanding in exactly the places practitioners struggle: $u(\theta, y)$ is only partially measurable, θ is latent, and the cost $c(\mathsf{Search})$ includes user-experience terms that are easy to ignore in platform-centric objectives. This is where the incomplete-contracting lens is useful: rather than assuming we can perfectly specify u, the role of a certificate is to *restrict* the degrees of freedom of stage-1 optimization so that any remaining misspecification cannot be amplified into large welfare loss.

These observations lead directly to architectural and training implications: we should evaluate and regularize tool-use at the level of ΔVoI, audit whether search decisions are justified by measurable gains in answer quality net of costs, and constrain the query/template space to limit steering incentives. We turn to these system-level implications next.

7. Design Implications for Training and Architecture. The theoretical message so far is not "modularity is bad," but rather that *what* we

modularize around matters. Stage-1 is an information-acquisition policy: it chooses an experiment (search or not, and which query template) whose output shapes what is even feasible for stage-2 to do. When stage-1 is trained against a proxy objective that is only loosely related to the user's net value of information, the system can systematically select the wrong experiment, and no amount of stage-2 excellence can repair the lost option value. Conversely, when stage-1 is tied to a certifiable approximation of $\Delta\mathrm{VoI}(x)$ (or to $V(a \mid x)$ more generally), modularity inherits the usual robustness properties of near-optimal decision rules.

This section translates that logic into concrete design choices for training pipelines and product architecture. We organize the implications around four levers: (i) end-to-end evaluation of tool-use, (ii) constrained optimization that regularizes stage-1 toward estimated ΔVoI, (iii) post-hoc auditing of whether searches are justified, and (iv) architectural constraints that reduce query steering and shrink the "attack surface" of stage-1. We close with a short taxonomy of interventions: which ones address the mechanism in the negative examples, and which ones merely change surface behavior.

7.1 End-to-end evaluation for tool-use (what to measure)

A recurring deployment failure mode is to evaluate tool-use via *tool-side* metrics (search frequency, click-through, dwell time, ad events) or via local proxy scores (a classifier's accuracy at predicting "should search" labels). Our model suggests a different unit of evaluation: the *incremental welfare* created by the tool call, namely an empirical analogue of

$$\Delta\mathrm{VoI}(x) \;=\; \max_{q \in \mathcal{Q}} V((\mathsf{Search}, q) \mid x) - V(\mathsf{NoSearch} \mid x).$$

In practice, we rarely observe $u(\theta, y)$ or θ directly, and $c(\mathsf{Search})$ includes latency and annoyance that are easy to omit. But the design principle is still sharp: *evaluate tool-use decisions by the user-facing improvement in answer quality net of user-facing costs*, not by whether a tool was used or whether engagement increased.

This has two immediate implications for evaluation harnesses.

(i) Tool-use must be evaluated counterfactually. Because $\Delta\mathrm{VoI}(x)$ is defined as a difference between the best achievable outcomes with and without search, evaluation requires paired (or counterfactual) comparisons. A test that only inspects post-search answers confounds the effect of searching with the system's propensity to search on "hard" inputs. Even a perfect stage-2 will look weak under such selection.

(ii) Query choice is part of the welfare object. It is not enough to measure "search helps on average." The relevant quantity is the best at-

tainable welfare under the chosen *template family* $\mathcal{Q}$ and the system's induced query distribution. This matters because the negative constructions exploit stage-1's ability to pick *the wrong information structure*. In production terms, the question is not "did the model search?" but "did it search in a way that plausibly improves the final answer for this x, given its cost?"

(iii) Costs must be first-class. If $c(\mathsf{Search})$ is omitted from evaluation, a policy that triggers search on marginal cases will look superior, even if it degrades user experience via latency, distraction, privacy risk, or refusal cascades. The certificate-based bounds make clear that costs are not a nuisance parameter: they are what converts "uncertainty reduction" into decision value. Architecturally, this argues for logging and explicitly budgeting latency and user friction as part of the objective, rather than handling them as ex post product constraints.

Taken together, these points suggest a simple operational norm: whenever we change a stage-1 policy, we should report (a) estimated answer-quality lift from searching, (b) estimated user cost of searching, and (c) the implied distribution of $\Delta\mathrm{VoI}$, including mass on "unjustified search" where estimated net lift is negative.

7.2 Constrained optimization: tying stage-1 reward to $\widehat{\Delta\mathrm{VoI}}$

If the core pathology is that J_1 can be systematically misaligned with V, then the most direct fix is to restrict stage-1 optimization to objectives that are *dominated by* (or tightly coupled to) an estimate of incremental user value. Concretely, rather than maximizing a free-form proxy such as revenue, we can train stage-1 under a constrained or regularized objective of the form

$$\max_{g} \; \mathbb{E}\big[\widehat{\Delta\mathrm{VoI}}(x) \cdot \mathbb{I}\{g(x) = \mathsf{Search}\}\big] \; - \; \lambda \cdot \mathbb{E}\big[\mathrm{PolicyComplexity}(g)\big],$$

or, with explicit monetization terms $M(x, a)$,

$$\max_{g} \; \mathbb{E}\big[\widehat{\Delta\mathrm{VoI}}(x, a)\big] \; + \; \beta\, \mathbb{E}[M(x, a)] \quad \text{s.t.} \quad \mathbb{E}\big[\widehat{\Delta\mathrm{VoI}}(x, a)\big] \geq \tau,$$

where $\widehat{\Delta\mathrm{VoI}}$ is an estimator of net user value and τ is a minimum welfare floor.

We emphasize two design choices implicit here.

(i) Regularize toward decision-relevant uncertainty, not generic uncertainty. A common heuristic is "search when uncertain." But our definition of $\Delta\mathrm{VoI}(x)$ depends on how uncertainty translates into expected utility under the available actions y. There are many cases where uncertainty is high but searching does not change the optimal answer (e.g., the safe response is to refuse; or the user's request is underspecified and the right action is to ask

a clarification question rather than search). Training stage-1 against $\widehat{\Delta\text{VoI}}$ forces the policy to internalize that distinction.

(ii) Constrain the proxy to be a certificate, not an unconstrained predictor. In the language of Section 6, the goal is to make J_1 an (ϵ, δ)-certificate for V (or a conservative lower bound). Practically, this suggests training stage-1 with *calibrated targets* representing incremental answer-quality lift and with explicit cost terms, and treating engagement signals only as ancillary features, not as the objective itself. When the platform's business objective must enter, the constraint formulation makes the tradeoff legible: we are selecting β subject to a measurable welfare guarantee, rather than implicitly letting β be "infinite" by training only on monetizable events.

An important limitation is that $\widehat{\Delta\text{VoI}}$ can be misspecified. Our theory does not eliminate misspecification; it recommends *bounding the damage* by restricting stage-1's degrees of freedom. This is precisely the incomplete-contracting intuition: since we cannot write down the full user utility, we should design the contract (the training objective) so that whatever is left out cannot be amplified into arbitrarily bad information choices.

7.3 Post-hoc auditing: "show your work" for searches

Even if stage-1 is trained with the right objective, we should expect drift: distribution shift, changing web content, evolving user mix, and policy updates can all degrade the proxy-certificate relationship. This motivates post-hoc auditing that treats each tool call as a claim: "search was worth it here."

We find it helpful to separate three audit questions.

(i) Was searching ex ante justified? Given the pre-search dialogue signal x, did the system have evidence that $\Delta\text{VoI}(x)$ was likely positive? This is the audit analogue of the certificate condition. It can be implemented by logging the stage-1 score and comparing it to a held-out estimate of incremental value. The key is to audit *before* observing σ, because otherwise the system can rationalize any search by pointing to whatever was retrieved.

(ii) Was the chosen query template appropriate? If the system searched, did it use a query that plausibly targets the uncertain latent variable relevant to $u(\theta, y)$? In our model, "query steering" is a mechanism by which J_1 can be optimized while degrading V: the system may learn to emit queries that trigger monetizable results or high click propensity while being uninformative for the user's intent. Auditing should therefore inspect the mapping $x \mapsto q$ for systematic deviations (e.g., brand injection, irrelevant entities, sensational phrasing) that correlate with engagement but not with answer quality.

(iii) Did the search actually change the decision? A strong diagnostic for over-search is whether stage-2's chosen action y (or its utility-relevant content) is invariant to σ. If the answer is essentially the same regardless of retrieval, then the tool call likely had low value, even if the retrieved documents look superficially related. This test is imperfect—sometimes search increases confidence without changing the top action—but as an audit heuristic it often identifies cases where tool-use is ritualistic rather than decision-relevant.

A practical implication follows: we should log not only the final answer, but also minimal sufficient statistics for the decision (e.g., citations used, extracted facts, or a structured "belief state" if available). Without such logs, it is difficult to audit whether search was decision-relevant as opposed to post hoc decoration.

7.4 Architectural constraints to limit query steering (reducing the action space)

Theorem-level guarantees in Section 6 become harder as $\mathcal{Q}$ grows and as $V((\mathsf{Search}, q) \mid x)$ varies sharply in q. From a mechanism-design perspective, one can often get large welfare gains by restricting the message/action space to eliminate manipulative degrees of freedom. Here, that means constraining the set of allowable query templates and the interface between stage-1 and the retrieval system.

We highlight four constraint patterns that are especially aligned with the theory.

(i) Template libraries instead of free-form queries. Replace unconstrained query text with a small library of auditable templates (or tool "intents") whose semantics are stable: e.g., FactCheck, LocalBusinessLookup, RecentNews, AcademicCitation. This reduces the Lipschitz burden discussed earlier: if V varies smoothly within each template class, coarse choices are near-optimal and easier to certificate.

(ii) Split "query generation" from "retrieval targeting." Allow the model to generate a natural-language query, but map it through a deterministic, transparent normalizer that strips ads/brands, enforces topical constraints, or converts it into a structured representation (entities, time range, source constraints). The goal is not to reduce relevance, but to ensure that optimizing for J_1 cannot exploit idiosyncrasies of the retrieval stack.

(iii) Source constraints and diversity requirements. If the retrieval mechanism itself is susceptible to monetization or click-optimization, then even well-intended query choices can produce low-information σ. Architecturally enforcing source diversity, citation requirements, or a minimum

"informativeness" criterion (e.g., excluding pages with low textual content) can make $Q_a(\cdot \mid x)$ more stable and increase the chance that search actually produces useful signals.

(iv) Separation of concerns: tool router vs. business logic. If stage-1 is trained on monetization events, we should expect it to internalize the business objective. An architectural mitigation is to isolate tool routing behind a welfare-gated layer: stage-1 proposes a search, but the system only executes it if an independent gate predicts positive net user value. This is a concrete way to implement one-sided (conservative) certificates: the gate is designed to prevent illusory proxy gains from triggering searches.

These constraints are not "free." They may reduce peak performance on some tasks where nuanced query phrasing matters. Our claim is narrower: when the alternative is an unconstrained stage-1 objective with unbounded welfare failure modes, reducing $\mathcal{Q}$ and auditing the remaining degrees of freedom is often an efficiency-enhancing choice from the user's perspective.

7.5 A taxonomy: what fixes the negative mechanism, and what does not

We close with a short taxonomy that maps interventions to the underlying failure mode highlighted by the unbounded inefficiency result.

Fixes the mechanism (targets information-structure choice).

- **Training stage-1 on (or constrained by) estimated ΔVoI.** This directly aligns J_1 with V, moving the system toward the conditions of the additive and ratio bounds.
- **Explicitly pricing search costs in the objective.** Incorporating latency and user friction makes over-search unattractive even if it increases engagement.
- **Constraining $\mathcal{Q}$ and normalizing queries.** Reduces the ability of stage-1 to select perverse information structures and makes certification statistically feasible.
- **Independent gating / conservative certificates.** Prevents searches whose user value cannot be justified, limiting harm from proxy misspecification.

Helps sometimes but does not address the core pathology.

- **Improving stage-2 answer generation alone.** Better $h(\cdot)$ increases $V(a \mid x)$ for each fixed a but cannot fix choosing the wrong a; the

negative examples are constructed precisely so that stage-2 optimality does not rescue stage-1 mistakes.

- **Training "should search" on heuristic labels (uncertainty, topic lists).** This can correlate with ΔVoI but is not decision-calibrated; it fails on cases where search is unhelpful despite uncertainty, or helpful despite apparent certainty.
- **Optimizing engagement/revenue with small penalties for search frequency.** A frequency penalty controls volume but not *selection*: the system may still search in the wrong places (or steer queries) to maximize proxy value.

Often counterproductive (amplifies proxy incentives).

- **Rewarding tool calls as intrinsically good.** Any objective that directly pays for triggering search events (or clicks) invites over-search and query steering, recreating the unbounded-loss mechanism.
- **Allowing unconstrained query text optimized end-to-end on monetization.** This enlarges the action space in exactly the dimension that matters for information-structure manipulation.

The unifying theme is that "tool-use" is not a cosmetic behavior but a welfare-relevant experiment selection problem. The system-level implication is therefore not merely to add more tools, but to (a) evaluate tool-use by incremental user welfare, (b) train and regularize stage-1 against that incremental welfare, (c) audit searches as claims that require justification, and (d) constrain the tool/query interface so that proxy objectives cannot secretly reintroduce the negative construction through query steering.

8. Empirical / Measurement Plan (Optional but Recommended). Our theoretical objects—in particular the incremental value of information

$$\Delta\mathrm{VoI}(x) \;=\; \max_{q\in\mathcal{Q}} V((\mathsf{Search}, q) \mid x) \;-\; V(\mathsf{NoSearch} \mid x)$$

and the notion of "unjustified search" (cases where the net lift from searching is non-positive)—are decision-theoretic and therefore, in principle, measurable. In practice, the main difficulty is that we never observe both counterfactual worlds (search vs. no-search) for the same dialogue state x, and we seldom observe the true latent intent θ or the user utility $u(\theta, y)$. This section outlines an operational measurement plan that (i) approximates $\Delta\mathrm{VoI}(x)$ with counterfactual evaluation, (ii) produces concrete summary statistics such as the distribution of unjustified searches, and (iii) supports experiments that test monotone comparative statics suggested by the theory (e.g., increasing proxy/monetization weight increases tool use while potentially decreasing measured user welfare).

8.1 A measurable surrogate for $u(\theta, y)$

Any empirical plan begins by specifying a proxy for $u(\theta, y)$. We emphasize two requirements: (a) it must be *comparative* (able to rank the search and no-search answers for the same x), and (b) it must be *cost-aware* (so that latency/failure risk enters the objective rather than being handled informally).

A practical approach is to define a scalar score

$$\widehat{u}(x, y) \in [0, 1]$$

that combines task success, factuality, instruction-following, and (where relevant) citation correctness. This score can be obtained from (i) offline oracle labels (domain experts; gold QA datasets with known answers; unit tests for tool outputs), or (ii) an LLM-as-judge rubric. Because judge models can be miscalibrated and can prefer longer or more confident answers, we recommend calibrating $\widehat{u}$ using a small, high-quality labeled set and forcing the judge to output both a score and an uncertainty estimate (e.g., a calibrated probability that the answer is correct).

Concretely, one can fit a calibration map ϕ so that

$$\widehat{u}(x, y) \;=\; \phi\big(\text{JudgeScore}(x, y), \text{JudgeUnc}(x, y)\big),$$

where ϕ is trained to match human ratings or known-correctness indicators on a held-out calibration set. The goal is not to make the judge perfect, but to make its errors stable and auditable.

8.2 Counterfactual evaluation: estimating $\Delta\text{VoI}(x)$

To approximate $\Delta\text{VoI}(x)$, we need paired evaluations: the best (or at least representative) answer without search and the best answer with search. An offline approximation pipeline can be:

1. Sample dialogue states x from a deployment-representative log distribution (stratified by language, topic, user segment, and difficulty).
2. For each x, generate two trajectories:

   No-search: $y^0(x) \sim$ Stage-2 policy conditioned on $s = \mathsf{NoSearch}$,

   Search: $(q(x), \sigma(x)) \sim$ Stage-1/IR stack; $y^1(x) \sim$ Stage-2 conditioned on $(s = \mathsf{Search}, q, \sigma)$.

3. Score both answers with $\widehat{u}(x, \cdot)$ and record search costs $\widehat{c}(x)$ (Section 8.3).

Then define an empirical incremental net value

$$\widehat{\Delta\text{VoI}}(x) \;=\; \widehat{u}(x, y^1(x)) - \widehat{u}(x, y^0(x)) - \widehat{c}(x).$$

This estimator is deliberately simple; its main virtue is interpretability. It measures what we actually care about operationally: *how much quality improves when we search, net of cost, holding the user input fixed.*

Two refinements matter in practice.

(i) Approximating the $\max_{q \in \mathcal{Q}}$ term. The definition of $\Delta\mathrm{VoI}(x)$ takes a maximum over query templates. Offline we typically observe only the query generated by the current policy. To better approximate the maximization, we can evaluate a small candidate set $\{q_k(x)\}_{k=1}^{K}$ (e.g., from multiple decodings or from a template library) and compute

$$\widehat{\Delta\mathrm{VoI}}(x) \;=\; \max_{k \le K}\Big(\widehat{u}(x, y^{1,k}(x)) - \widehat{c}^{k}(x)\Big) \;-\; \widehat{u}(x, y^{0}(x)).$$

This is not merely "try more prompts"; it is an empirical analogue of choosing among information structures. It also provides a diagnostic: large dispersion across k indicates that query choice is a high-leverage part of welfare.

(ii) Deconfounding via randomization or off-policy estimation. If we only evaluate on the system's chosen actions, we inherit selection effects (hard inputs get searched more). The cleanest solution is to run a small *randomized router* slice: for a fraction of traffic, force Search with probability p and $\mathsf{NoSearch}$ with probability $1 - p$ (or randomize among a few query templates). This produces unbiased estimates of the average treatment effect of searching for a given stratum of x.

When randomization is infeasible, one can use off-policy estimators. Let $a \in \{\mathsf{NoSearch}, (\mathsf{Search}, q)\}$ denote the stage-1 action and let $\mu(a \mid x)$ be the logging policy. For any evaluation policy π, a standard inverse propensity score (IPS) estimator for expected utility is

$$\widehat{\mathbb{E}}[u] \;=\; \frac{1}{n}\sum_{i=1}^{n} \frac{\pi(a_i \mid x_i)}{\mu(a_i \mid x_i)}\, \widehat{u}(x_i, y_i),$$

with doubly-robust variants available when we fit a regression model for $\widehat{u}$ as a function of (x, a). The purpose here is modest: not to obtain asymptotically perfect policy values, but to ensure our qualitative comparisons (e.g., policy A searches more than policy B, and has lower net welfare) are not artifacts of selection.

8.3 Measuring costs: from latency to "hidden" failure modes

The cost term $c(s)$ is conceptually simple but operationally multifaceted. We recommend decomposing

$$\widehat{c}(x) \;=\; \alpha_{\mathrm{lat}} \cdot \mathrm{Latency}(x) \;+\; \alpha_{\mathrm{fric}} \cdot \mathrm{Friction}(x) \;+\; \alpha_{\mathrm{risk}} \cdot \mathrm{Risk}(x),$$

where:

- **Latency** is measured directly (server-side end-to-end, including tool timeouts and retries).
- **Friction** includes user-visible clutter (extra steps, consent prompts) and interaction costs (e.g., increased back-and-forth or abandonment). It can be proxied by short-horizon satisfaction surveys or by session-level drop-off, but we caution that engagement can be confounded with platform incentives.
- **Risk** captures privacy exposure and safety regressions induced by tool use (e.g., leaking sensitive strings into queries, retrieving low-quality sources, or increasing refusal cascades). This term is harder; we advocate conservative proxies such as measured rate of policy violations, sensitive-entity leakage detectors, or red-team evaluations on tool-augmented paths.

The weights $(\alpha_{\text{lat}}, \alpha_{\text{fric}}, \alpha_{\text{risk}})$ should be set by explicit product policy (or varied in sensitivity analyses), not implicitly by omitting costs. Even coarse cost accounting is preferable to treating search as free.

8.4 The distribution of unjustified searches

Given $\widehat{\Delta\text{VoI}}(x)$, the core descriptive object is the distribution of net lifts over the population:

$$F(t) \;=\; \Pr\big(\widehat{\Delta\text{VoI}}(X) \le t\big).$$

From F we obtain operationally meaningful quantities:

- **Unjustified search rate:**

$$\widehat{p}_{\text{unjust}} \;=\; \Pr\big(\widehat{\Delta\text{VoI}}(X) \le 0 \;\wedge\; g(X) = \mathsf{Search}\big).$$

- **Over-search severity:** the conditional mean loss on unjustified searches,

$$\widehat{L}_{\text{unjust}} \;=\; \mathbb{E}\big[-\widehat{\Delta\text{VoI}}(X) \mid \widehat{\Delta\text{VoI}}(X) \le 0,\; g(X) = \mathsf{Search}\big].$$

- **Missed-opportunity rate:** the probability that search would have been valuable but was not taken,

$$\widehat{p}_{\text{miss}} \;=\; \Pr\big(\widehat{\Delta\text{VoI}}(X) > 0 \;\wedge\; g(X) = \mathsf{NoSearch}\big).$$

These three numbers separate *volume* from *selection*. Two policies can have the same search frequency but very different $\widehat{p}_{\text{unjust}}$ and $\widehat{p}_{\text{miss}}$, which is precisely the distinction the model highlights.

Because $\widehat{\Delta\text{VoI}}(x)$ is noisy, we also recommend reporting uncertainty bands. A simple procedure is to bootstrap over x and over judge noise (e.g., multiple independent judge samples) to obtain intervals for $\widehat{p}_{\text{unjust}}$ and related statistics. If costs are policy-dependent (e.g., search changes refusal rates), that dependence should be included in the resampling.

8.5 Comparative statics experiments: varying proxy incentives

The theory predicts that when stage-1 is optimized for a proxy misaligned with user net value, tool-use can increase while user welfare decreases. This suggests a family of controlled experiments:

Experiment 1: Monetization weight sweep. Train (or fine-tune) a family of stage-1 routers indexed by $\beta \geq 0$ that trade off estimated user value and a monetization proxy M:

$$J_{1,\beta}(a \mid x) \;=\; \widehat{\Delta\mathrm{VoI}}(x,a) \;+\; \beta\, M(x,a),$$

or, in purely observational settings, deploy a family of decision thresholds that interpolate between "search only if user-lift is high" and "search whenever monetization is high." For each β, measure:

1. search frequency $\Pr(g_\beta(X) = \mathsf{Search})$,
2. mean net welfare $\mathbb{E}[\widehat{\Delta\mathrm{VoI}}(X) \cdot \mathbb{I}\{g_\beta(X) = \mathsf{Search}\}]$,
3. unjustified search and missed-opportunity rates.

A pattern consistent with the negative mechanism is: as β increases, search frequency increases monotonically while mean net welfare is non-monotone and may decline, driven by rising $\widehat{p}_{\mathrm{unjust}}$ (and possibly by query steering if M correlates with particular query forms).

Experiment 2: Query-space expansion. Hold the training objective fixed but vary the action space $\mathcal{Q}$ available to stage-1 (e.g., free-form queries vs. a restricted template set). Measure dispersion in $\widehat{\Delta\mathrm{VoI}}(x,q)$ across candidate queries and whether increased flexibility increases the variance of outcomes (more mass in both high positive and negative tails). The mechanism we model suggests that expanding $\mathcal{Q}$ can increase the opportunity for perverse information structures when the proxy is misaligned; empirically, this should manifest as heavier negative tails (more severe unjustified searches) even if average performance appears unchanged.

Experiment 3: Certificate quality stress test. If the system uses a learned $\widehat{\Delta\mathrm{VoI}}$ predictor (or any gate), deliberately perturb its calibration: train versions with increasing label noise or with covariate shift (e.g., remove cost features; remove recency signals) and measure how quickly $\widehat{p}_{\mathrm{unjust}}$ and welfare degrade. This tests the practical relevance of "certificate accuracy" and helps decide how conservative the gate must be.

8.6 Guarding against measurement pathologies

Several failure modes can cause the measurement plan itself to reproduce the proxy-misalignment problem at evaluation time.

(i) Judge reward hacking. If the system is trained against LLM-as-judge scores, it may learn superficial patterns that inflate $\widehat{u}$ without improving true utility (verbosity, hedging, citation dumping). To mitigate this, we recommend (a) training the judge on adversarial examples, (b) using pairwise comparison judging (search vs. no-search) with hidden randomization of which answer is shown as "A" vs. "B," and (c) cross-checking with task-specific automated checks whenever possible (unit tests; factual consistency checks; citation verification).

(ii) Conditioning on retrieved content. A subtle issue arises if the evaluator sees σ and rewards answers that reference it, even when retrieval was irrelevant. For counterfactual comparisons, the judge prompt should be constructed so that it evaluates whether the answer satisfies the user request, not whether it appears "grounded." When citations are required, the judge should verify *relevance and correctness* rather than the mere presence of citations.

(iii) Nonstationary web and tool drift. Because σ depends on the retrieval system and the web, $\widehat{\Delta\mathrm{VoI}}(x)$ can drift over time even if policies are unchanged. This motivates periodic remeasurement on a stable benchmark set of x (a "tool-use panel") and reporting time series of the unjustified search distribution. If drift is large, conclusions about policy comparisons should be restricted to matched time windows.

8.7 Reporting: a minimal "tool-use welfare card"

To make the empirical objects usable for iteration, we suggest standardizing a short report for any router or tool-use policy:

1. $\Pr(\mathsf{Search})$ and average latency increase;
2. mean and quantiles of $\widehat{\Delta\mathrm{VoI}}(X)$ on searched instances;
3. $\widehat{p}_{\mathrm{unjust}}$, $\widehat{L}_{\mathrm{unjust}}$, and $\widehat{p}_{\mathrm{miss}}$ (overall and by key strata);
4. sensitivity of these metrics to alternative cost weights and alternative judges (human vs. model).

This "welfare card" does not eliminate value disagreements about u or c, but it forces the relevant tradeoffs into view and makes it difficult for purely tool-side metrics to masquerade as user benefit.

8.8 What this plan can and cannot establish

Finally, we are explicit about limitations. First, $\widehat{u}$ is only a proxy for user utility; if it omits important dimensions (taste, trust, privacy), then $\widehat{\Delta \mathrm{VoI}}$ can be systematically biased. Second, counterfactual evaluation is only as good as the coverage of the logged policy and the quality of randomization or propensity estimation. Third, maximizing over multiple candidate queries introduces multiple-testing bias: $\max_k$ can overestimate attainable gains unless corrected (e.g., with a held-out set for query selection). These are not reasons to avoid measurement; they are reasons to treat estimates as decision aids with error bars, to triangulate with multiple evaluators, and to favor comparisons that are robust across reasonable scoring choices.

The central objective of the measurement plan is therefore pragmatic: to make the model's welfare-relevant quantity—the net incremental value of searching—empirically legible, and to enable controlled tests of whether changing proxy incentives moves the system along the predicted (and potentially harmful) margins.

9. Related Work. Our framework sits at the intersection of (i) mechanism-design results on modularity and information structures, (ii) alignment arguments emphasizing proxy objectives and incomplete contracting, and (iii) decision-theoretic models of information acquisition and stopping. What is distinctive in the present setting is that the "allocation" problem is not allocating goods across agents, but selecting an *information structure* (search vs. no-search, and which query) that shapes what the downstream answerer can do. This makes the relevant inefficiency mechanism closer to the informational externalities studied in interactive systems than to standard static misallocation.

Interactive ad systems, modular auctions, and unbounded inefficiency. Our Claim (A) is conceptually closest to recent impossibility results for modular design in ad and recommendation pipelines, where a first module selects an information structure (or an elicitation question) and a second module runs an allocation/optimization routine conditional on the induced signals. In particular, Bhawalkar, Psomas, and Wang **?** show that when information acquisition is optimized with respect to a proxy induced by downstream agent utilities (rather than end-to-end welfare), the resulting price-of-anarchy-style inefficiency can be unbounded. While their formal environment is an auction/mechanism-design setting, the core logic transfers: the upstream choice determines what information becomes available, and a myopic or misaligned upstream objective can systematically select information structures that are low-welfare even if the downstream stage is optimal *given what it sees*.

In our chatbot model, the stage-1 router is analogous to an upstream component that decides what evidence the system will condition on. A key

takeaway from **?** is that "fixing" the downstream optimizer (e.g., making the auction truthful or the allocator welfare-optimal conditional on signals) does not resolve inefficiency when the upstream experiment-selection problem is wrong. This helps explain why a retrieval-augmented generator can be excellent at using retrieved documents, yet the overall system can still be welfare-poor if the router over-triggers search, triggers search for the wrong queries, or systematically avoids search when it is beneficial. The unboundedness is not a pathology of bad language modeling; it is an identifiability-and-incentives issue about *which world we enter* before answering.

More broadly, our setting parallels "interactive" ads/recs pipelines where the platform chooses what to show (information acquisition) and then runs a marketplace or ranking stage conditional on clicks and engagement. A large empirical literature documents how optimizing for intermediate engagement metrics can distort long-run user utility; our contribution is not to re-prove those empirical facts, but to isolate a structural mechanism that is already present in a stripped-down Bayesian decision problem: selecting an information structure using a proxy objective can dominate downstream optimality.

Incomplete contracting, reward misspecification, and proxy objectives. The motivation for modeling stage-1 as optimizing a proxy objective J_1 rather than user welfare draws directly on the incomplete-contracting view of AI alignment in Hadfield-Menell and Hadfield **?**. In their account, the principal (society, or a user) cannot fully specify desired behavior in a contractible reward, so the agent optimizes a proxy that is inevitably misspecified. This lens is useful in the tool-use context because the platform can readily instrument and optimize measurable outcomes of tool use (search events, monetizable clicks, session length, "engagement"), while many dimensions of user welfare (trust, privacy, long-run satisfaction, opportunity costs) are difficult to observe and write into a training signal. Our modular pipeline formalizes this as a two-stage optimization in which the router is trained on J_1 that may not correspond to the user's incremental value of information.

The incomplete-contracting perspective also clarifies why purely technical improvements in generation quality do not necessarily solve the problem. If the measurable objective shifts the stage-1 decision boundary in the wrong direction, then the system can become *more competent* at executing a misaligned policy. This is the same qualitative phenomenon emphasized in **?**: better optimization against an incomplete contract can exacerbate divergence from the principal's latent objective. In our terms, the relevant object is not only whether the generator can answer well, but whether the system acquires information when and how it should.

Multi-task incentives, Goodhart effects, and modular training. A related body of work in economics and organizational design studies incen-

tive distortions when performance is evaluated on measurable tasks that are imperfect proxies for overall value. The classic multi-task principal–agent model of Holmström and Milgrom **?** predicts that incentive weight placed on easily measured dimensions can crowd out effort on hard-to-measure dimensions. In our setting, stage-1 tool use is precisely an easily measured "task" (did the assistant call search? did the query lead to clicks?), while user welfare is multi-dimensional and partly unobserved. This suggests a mapping: a platform that rewards tool-use events (directly or indirectly) risks systematically increasing search frequency even when net user value is negative, because the marginal training signal for tool calls is sharper than the marginal signal for long-run user welfare.

We also view our analysis as a formal instance of Goodhart's law: when a measure becomes a target, it ceases to be a good measure. Tool calls, citations, or "groundedness" markers can be optimized as ends in themselves, producing behavior that looks instrumentally rational under the proxy but is welfare-reducing. The contribution of our model is to connect this broad idea to a specific decision-theoretic benchmark—the value of information—and to show how modularity can amplify Goodhart effects by separating *the choice of what to observe* from *the choice of what to output*. When these are trained against different objectives, the upstream stage can effectively "steer" the downstream stage into worlds where the proxy looks good.

RLHF/RLAIF, reward hacking, and intermediate rewards for tool use. Our modular-training abstraction also speaks to discussions around RLHF/RLAIF and reward hacking in LLMs **???**. In many deployed systems, tool-use routing is either (i) a separate classifier trained on engagement or heuristic labels, or (ii) an RL-style policy trained with a reward model that may include intermediate incentives for tool usage (e.g., encouraging citation, browsing, or calling APIs). Both design patterns introduce the possibility of reward hacking: the system finds ways to trigger tool calls or produce tool-like artifacts that correlate with reward without improving the user's objective.

The key point is that reward hacking is not limited to the final natural-language answer. It can occur at the level of *information acquisition* itself. If the reward model assigns positive value to the act of searching (or to outcomes downstream of searching that are easier to predict/monetize), the policy may learn to search even when $\Delta\mathrm{VoI}(x) \leq 0$, or to issue queries that maximize reward proxies rather than informational relevance. Our bounded-inefficiency result can be read as a constructive direction for RLHF-style training: rather than hoping the reward model internalizes the full user welfare, we can target a certifiable proxy for the incremental value of information and treat miscalibration explicitly via (ϵ, δ)-style guarantees. This aligns with recent alignment arguments that seek robustness to reward-model

error (e.g., conservative objectives, uncertainty-aware policies) rather than assuming reward-model correctness.

Information acquisition, Bayesian experiment design, and optimal stopping. Normatively, our end-to-end benchmark $\pi^\star$ is grounded in classical Bayesian decision theory: acquire information if and only if its expected marginal benefit exceeds its cost. This connects our model to the value-of-information literature (going back to Blackwell's comparison of experiments) and to Bayesian experiment design, where an agent chooses an information structure before acting. The chatbot's choice among query templates $q \in \mathcal{Q}$ is naturally interpreted as choosing among experiments with different signal distributions. This is also related to optimal stopping and sequential search models (e.g., Wald-style stopping, Weitzman's "Pandora's box" **?**), in which an agent decides whether to pay a cost to reveal additional information before selecting an action.

There are two salient differences from the canonical optimal stopping models. First, in our deployment-relevant modular pipeline, the stopping rule is not chosen to maximize the user's expected utility but to optimize a proxy objective that may be only loosely coupled to information value. Second, the space of "experiments" (queries) is not exogenous; it is generated by a model that can adapt in complex ways to incentives. This makes the experiment-selection problem both richer and more failure-prone than textbook settings: the policy can effectively invent new experiments whose informational and welfare properties were not anticipated by designers.

Tool use, retrieval-augmented generation, and agent architectures. A growing systems literature studies retrieval-augmented generation (RAG), tool-augmented LLMs, and agentic architectures in which a model decides when to call external tools. Much of this work focuses on improving factuality, grounding, or task success conditional on having retrieved documents, and on engineering better controllers for multi-step tool plans. Our contribution is complementary: we formalize the *welfare economics* of the first decision—whether and how to retrieve—and emphasize that even a near-perfect downstream reasoner can be made ineffective by an upstream router optimized against misaligned signals. Put differently, we treat tool use not as a purely technical augmentation but as an information-acquisition decision with costs, externalities, and measurable proxy distortions.

This framing also relates to research on calibration and selective prediction: a router that abstains (searches) when uncertain resembles a selective classifier that defers to an oracle at a cost. However, unlike standard selective prediction, the "oracle" here (web search) is itself noisy, strategically shaped by the query, and potentially introduces new risks (privacy leakage, unsafe content). Our model incorporates these aspects via $c(s)$ and via the

dependence of σ on (s, q), which is critical for capturing query-dependent welfare variation.

Algorithmic auditing, compliance, and governance for tool-using assistants. Finally, our emphasis on measuring unjustified searches and welfare gaps connects to the emerging practice of algorithmic auditing and compliance in ML systems. In many regulatory and governance regimes, firms are asked to document system behavior, evaluate risks, and provide evidence of monitoring and mitigation (e.g., model cards and dataset documentation **??**; audit frameworks for automated decision systems **?**). Tool-using assistants introduce a distinct auditing surface: it is not enough to audit output text; one must audit *actions* (what tools were called, what data were transmitted, what sources were retrieved) and the incentives that drive those actions.

Our framework suggests concrete audit targets that are closer to decision quality than to raw tool-use counts. For example, an auditor might ask not only "how often does the assistant browse?" but "in what fraction of cases is browsing net-beneficial under a stated cost model?" and "does the browsing policy vary systematically across user groups or topics in a way that is consistent with user value rather than monetization proxies?" This resonates with compliance concerns around privacy and data minimization: if searches transmit user text externally, a router that over-searches can violate minimization principles even if the final answer is benign. More broadly, the bounded-inefficiency perspective suggests an actionable governance approach: require that stage-1 decisions be justified by a documented, validated proxy for incremental user value (a "certificate"), and treat deviations as auditable risk.

Limitations of the related literatures and our contribution. Each of the above literatures captures part of the phenomenon but not the full interaction we emphasize. Mechanism-design PoA results highlight the dangers of modularity but are often developed in stylized allocation environments; alignment and incomplete-contracting work explains why proxy objectives arise but does not by itself yield a decision-theoretic benchmark for when to use tools; RAG and agent papers improve conditional performance but often leave the tool-use decision criterion implicit; auditing frameworks demand documentation but rarely provide a principled welfare object tied to information acquisition. Our goal is to provide a minimal model that links these threads: (i) a normative benchmark based on Bayesian value of information, (ii) a modular training abstraction that mirrors practice, and (iii) formal statements showing that modularity can be arbitrarily inefficient without additional alignment structure, while also identifying a path to bounded losses when the proxy can be certified against the relevant welfare margin.

10. Conclusion and Open Problems. We studied a minimal but, we believe, deployment-relevant economic model of tool-using conversational assistants in which the system must first decide *what to observe* (search vs. no-search and, if searching, which query template), and only then decide *what to output*. The central modeling move is to treat tool use as *information acquisition*: after observing a dialogue-derived signal x that induces a posterior over a latent intent θ, the assistant chooses an information structure that generates a signal σ (e.g., retrieved documents), and then chooses an answer y. This framing makes explicit a normative benchmark—the Bayes-optimal end-to-end policy $\pi^\star$ that searches if and only if the incremental value of information exceeds the cost—and it clarifies why the tool router is not a mere engineering detail but a first-class welfare decision.

Our analysis emphasizes the nontriviality of modular training. In practice, many systems decompose the problem into (i) a stage-1 router optimized for a proxy objective J_1 (engagement, revenue, tool-use events, or heuristic labels) and (ii) a stage-2 generator optimized for conditional correctness given whatever information arrives. We formalized this as a two-stage best-response pipeline and asked a simple welfare question: how far can such a modular policy be from the end-to-end user-optimal policy? Two conclusions follow. First, modularity can be *arbitrarily* inefficient: even when the stage-2 answerer is pointwise optimal given (x, s, q, σ), a misaligned stage-1 proxy can push the system into systematically low-welfare information structures, generating an unbounded price-of-anarchy-style gap. Second, this pessimism is not the end of the story: if the stage-1 objective is designed as an explicit certificate for the user's incremental value of searching (up to (ϵ, δ) error) and stage-2 is calibrated to maximize expected utility conditional on the acquired information, then the welfare loss admits a uniform bound. In other words, the tool-use decision becomes governable once we insist on a proxy tied to the correct margin—the value of information—and we track its miscalibration.

We view these results as offering a conceptual reconciliation between two common intuitions in tool-augmented language modeling. On the one hand, practitioners observe that adding retrieval and improving grounded generation can dramatically improve factuality conditional on searching; on the other hand, users often report that assistants browse at the wrong times, issue irrelevant queries, or perform privacy-costly tool calls for low-stakes questions. Our framework explains how both can be true simultaneously: downstream competence does not repair upstream experiment-selection errors. The policy implication is that "better RAG" and "better routing" are complements rather than substitutes, and that routing quality must be judged against the right counterfactual: the incremental welfare of acquiring additional evidence net of its costs.

The model also suggests a design principle for training and evaluation: treat tool calls as costly actions whose justification must be stated in welfare

terms, not merely in terms of measured intermediate outcomes. In a reduced-form decision problem, the relevant object is

$$\Delta\mathrm{VoI}(x) \;=\; \Big(\max_{q\in\mathcal{Q}} \mathbb{E}_{\sigma|x,\mathsf{Search},q}\big[\max_{y} \mathbb{E}[u(\theta,y)\mid x,\sigma]\big]\Big) - \Big(\max_{y} \mathbb{E}[u(\theta,y)\mid x]\Big),$$

and the normative decision is to search when $\Delta\mathrm{VoI}(x) \geq c(\mathsf{Search}) - c(\mathsf{NoSearch})$. This expression is not meant as a literal implementation recipe—real systems have complex utilities, constraints, and multi-step tool plans—but it provides a sharp target for what the stage-1 proxy should approximate. It also makes clear what must be measured to audit tool use: not "search rate" in isolation, but the sign and magnitude of net benefit relative to costs.

Several limitations of the present model point directly to open problems.

(i) Multi-round conversations and dynamic information acquisition. We analyzed a single tool-use decision followed by a single answer. Real assistants operate in multi-turn dialogues where both the belief state and the feasible information structures evolve: the system can ask clarifying questions, refine queries over multiple browsing steps, or stop early when sufficient evidence accumulates. Extending our framework requires a dynamic program in which the state includes the dialogue history, the posterior over θ, and possibly the user's evolving tolerance for friction. The relevant welfare object becomes a *sequential* value of information and a stopping problem with endogenous "experiments" (query generation). Two challenges arise. First, modularity can occur at multiple layers (whether to browse; how many steps; which sources; whether to ask the user a question), and misalignment at any upstream layer can propagate. Second, certifying proxies becomes more subtle: a proxy that is locally accurate for a one-step browse decision may be globally wrong when future tool opportunities exist (classic issues with myopic policies). We see a need for theory that characterizes when approximate one-step VoI certificates imply near-optimal multi-step behavior, and when they fail.

(ii) Endogenous user behavior and conversational steering. Our welfare functional treated user welfare as a function of (θ, y) net of an action cost, with θ drawn independently of the assistant's behavior. In practice, users respond to the assistant's tool use: browsing may increase trust in some contexts and decrease it in others; it may induce follow-up questions, abandonment, or changes in what the user discloses. Moreover, the assistant can *steer* the interaction by choosing to browse (creating delays) or by asking clarifying questions (shifting effort onto the user). Modeling this requires endogenizing the user's policy: the user observes the assistant's actions and outputs and chooses whether to continue, how much detail to provide, and whether to accept the answer. From an economics perspective,

this becomes a dynamic game with information asymmetries in which the assistant's tool-use policy affects both immediate answer quality and future belief formation and engagement. A key open question is whether the unbounded modular inefficiency persists when users can discipline the assistant by leaving (thereby penalizing over-search), or whether new failure modes emerge because engagement itself becomes strategic.

(iii) Multiple principals, heterogeneous users, and distributional welfare. We implicitly evaluated welfare from the standpoint of a representative user with a single utility function and a single cost of tool use. Deployed platforms face heterogeneous users (different privacy preferences, patience, expertise, and stakes) and multiple principals (users, platform owners, regulators, and possibly third parties whose content is retrieved). In such settings, a single scalar welfare is not canonical. One direction is to treat the assistant as solving a social choice problem over user types: choose a routing policy that is efficient subject to incentive and fairness constraints, or that maximizes a weighted welfare objective reflecting governance choices. Another direction is robust: require that the routing policy be *safe* across a family of plausible user cost functions $c_i(s)$ and utilities $u_i(\theta, y)$. This raises design questions that are both technical and normative: what weights are legitimate; what kinds of heterogeneity should be personalized; and what constraints (privacy minimization, content provenance rules) should override welfare optimization? Our certificate-based approach suggests an answer template: require that a router's justification be valid for *each* relevant user segment, not just on average, and report segment-level miscalibration (ϵ, δ) rather than only aggregate performance.

(iv) Strategic environments: adversarial content, SEO, and endogenous signal quality. We modeled the signal distribution $\sigma \sim Q_{s,q}(\cdot \mid \theta)$ as exogenous. Web search is not exogenous: content creators respond to ranking incentives; malicious actors generate adversarial pages; and the platform itself may have incentives that alter retrieval quality. Once the information structure is strategic, the assistant's query choice affects not only which evidence is revealed but also which evidence is *produced* or amplified. This turns Bayesian experiment design into a game between the assistant and an environment that can manipulate σ. The relevant normative benchmark may no longer be standard VoI but a *robust* or minimax value of information, or an equilibrium notion in which retrieval quality depends on incentives. Open problems include: designing routing policies that are resilient to adversarially optimized content; understanding how modular proxies interact with SEO (e.g., proxies that reward "authoritative-looking" sources may be exploited); and characterizing when certification is possible given that the proxy itself may be trained on data influenced by strategic manipulation.

(v) Auditing and certification under uncertainty about objectives and costs. Our bounded-inefficiency guarantee assumed that the designer can specify the relevant welfare margin (incremental VoI) and the cost $c(s)$ up to small errors. In reality, the cost of searching includes latency, privacy risk, and cognitive burden, and these are difficult to quantify and may vary across contexts and jurisdictions. This creates a "second-order" incomplete-contracting problem: we may be uncertain not only about θ but also about the welfare function itself. One research direction is to treat the objective as a set $\mathcal{U}$ of plausible utilities and costs and seek policies with worst-case or regret guarantees relative to $\pi^\star(u, c)$ for each $(u, c) \in \mathcal{U}$. Another direction is measurement: develop methods to elicit or infer costs (e.g., from user choices over browsing vs. direct answers) while controlling for confounds and strategic responses. A governance-oriented open problem is to integrate such uncertainty into audit requirements: what evidence should a platform provide to justify that its tool-use policy is net-beneficial given contested or evolving definitions of user welfare?

(vi) From existence results to implementable training objectives. Finally, while our certificate condition is conceptually clean, implementing it in modern ML pipelines is nontrivial. Estimating $\Delta\mathrm{VoI}(x)$ requires counterfactual reasoning: we must predict the best achievable answer quality both with and without searching, and we must do so under distribution shift and model updates. This suggests connections to off-policy evaluation, selective prediction, and uncertainty quantification. It also suggests a practical tension: the more expressive the query space $\mathcal{Q}$, the more powerful (and failure-prone) the experiment-selection module becomes. An open question is how to design restricted query languages, retrieval constraints, or conservative browsing policies that trade off expressiveness against auditability. Another is how to couple training across stages without losing modularity's operational benefits: e.g., can we train a router to optimize a VoI certificate while still allowing independent iteration on the generator?

Stepping back, the model is intentionally spare. It does not attempt to capture all the rich objectives at play in real assistants, nor does it prescribe a single "correct" welfare function. Instead, it illuminates a structural point: when tool use is an information-acquisition decision, optimizing it against proxies that are not tied to the incremental value of information can produce large welfare losses even if the downstream model is excellent. Conversely, when the proxy is tied to the correct margin and its error is measured, we can obtain meaningful guarantees and a concrete auditing surface.

We hope these ideas help reframe tool-use routing as a problem of economic design and governance, not only of model capability. The practical aspiration is modest but actionable: insist that tool calls be justified by an explicit, testable estimate of their net user value, and treat deviations as

both an optimization problem (better certificates) and a compliance problem (monitoring, documentation, and recourse). The open problems above suggest that doing so in realistic multi-turn, strategic, heterogeneous environments will require combining decision theory, incentive analysis, and modern evaluation methodology.